



GRADE 12 EXAMINATION
NOVEMBER 2008

ADVANCED PROGRAMME MATHEMATICS

MARKING GUIDELINES

Time: 3 hours

300 marks

These marking guidelines are prepared for use by examiners and sub-examiners, all of whom are required to attend a standardisation meeting to ensure that the guidelines are consistently interpreted and applied in the marking of candidates' scripts.

The IEB will not enter into any discussions or correspondence about any marking guidelines. It is acknowledged that there may be different views about some matters of emphasis or detail in the guidelines. It is also recognised that, without the benefit of attendance at a standardisation meeting, there may be different interpretations of the application of the marking guidelines.

MODULE 1 CALCULUS AND ALGEBRA**QUESTION 1**

1.1 (a) $(x-1)(x^2+x+1)✓$ (1)

(b) $x=1✓$ (5)

or $x^2+x+1=0✓$

$$\therefore x = \frac{-1 \pm \sqrt{1-4}}{2}$$

$$\therefore x = \frac{-1 + \sqrt{3i}}{2} \quad \text{or} \quad \frac{-1 - \sqrt{3i}}{2} \quad ✓✓✓$$

1.2 $(x-1-\sqrt{2})(x-1+\sqrt{2})✓✓$

$$= x^2 - x + \sqrt{2}x - x + 1 - \sqrt{2} - \sqrt{2}x + \sqrt{2} - 2 \quad ✓✓✓ \text{ or}$$

$$(x-1)^2 - (\sqrt{2})^2$$

$$= x^2 - 2x - 1 \text{ is a factor. } ✓✓$$

By inspection:

$$x^4 - 2x^3 + 4x^2 - 10x - 5 = (x^2 - 2x - 1)(x^2 + 5) = (x^2 - 2x - 1)(x - \sqrt{5}i)(x + \sqrt{5}i) \quad ✓✓✓$$

(10)

16 marks

QUESTION 2

2.1 (a) $\log 2x - \log(x-20) = 1✓$

$$\therefore \log \frac{2x}{x-20} = 1 \quad ✓✓$$

$$\therefore \frac{2x}{x-20} = 10$$

$$\therefore 2x = 10x - 200$$

$$\therefore 8x = 200$$

$$\therefore x = 25✓$$

(4)

(b) $e^x = 5e^x - 5✓$

$$\therefore 4e^x = 5✓$$

$$\therefore e^x = 1,25✓$$

$$\therefore x = \ln 1,25 = 0,223...✓$$

(4)

(c) Let $k = |x|✓$

$$(k-6)(k+2) = 0✓✓$$

$$|x| = 6 \quad ✓✓ \quad |x| \neq -2✓$$

$$\therefore x = \pm 6$$

(6)

$$2.2 \quad 107 = 10 \log \left(\frac{L}{10^{-16}} \right) \quad \checkmark \checkmark$$

$$10,7 = \log L - \log 10^{-16} \quad \checkmark \checkmark$$

$$\therefore -5,3 = \log L \quad \checkmark \checkmark$$

$$\begin{aligned} \therefore L &= 10^{-5,3} \\ &= 5,01 \times 10^{-6} \end{aligned}$$

(6)

20 marks**QUESTION 3**

Prove: $x^{k+1} - y^{k+1}$ $\checkmark \checkmark$ is divisible by $x - y$.

$$x^k \cdot x - y^k \cdot y \quad \checkmark \checkmark$$

$$= (p(x - y) + y^k)x - y^k \cdot y \quad \text{from Step 2} \quad \checkmark \checkmark$$

$$= (px(x - y) + xy^k) - y^k \cdot y \quad \checkmark \checkmark$$

$$= px(x - y) + y^k(x - y)$$

$$= (x - y)(px + y^k) \quad \checkmark \checkmark$$

So we have shown that provided that $x^k - y^k$ is divisible by $x - y$ then so is $x^{k+1} - y^{k+1}$.

But since the statement is true for $n = 1$, then by the argument above it is true for $n = 2$, and hence $n = 3$ and so on for all natural values of n . $\checkmark \checkmark$

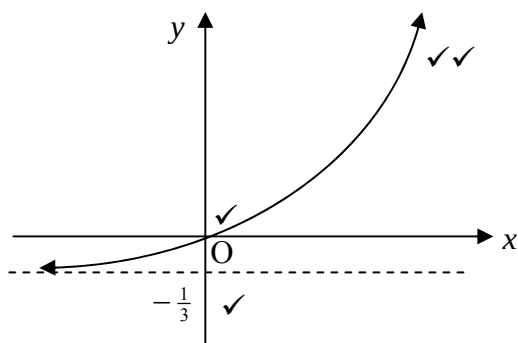
12 marks

QUESTION 4

4.1 (a) $f(x) = \frac{e^x(e^x - 1)}{3e^x}$

$$\therefore f(x) = \frac{1}{3}e^x - \frac{1}{3} \checkmark \checkmark \quad (2)$$

(b)



(4)

4.2 At Q, $\log(2x + 3) = 1 \checkmark$

$$\therefore 2x + 3 = 10$$

$$\therefore x = \frac{7}{2} \quad \checkmark \checkmark$$

At P, $\log(2x + 3) = -1$

$$\therefore 2x + 3 = \frac{1}{10} \quad \checkmark \checkmark \checkmark$$

$$\therefore x = -\frac{29}{20}$$

(6)

12 marks

QUESTION 5

5.1 $3 - a(1)^2 = -4(1) + 5 \checkmark \checkmark \checkmark$
 $\therefore a = 2 \checkmark \checkmark$

(5)

5.2 $f(x) = \begin{cases} 3 - 2x^2 & \text{if } x \geq 1 \\ -4x + 5 & \text{if } x < 1 \end{cases}$

$$\lim_{h \rightarrow 0^-} f'(x) = -4 \checkmark \checkmark \checkmark \quad \lim_{h \rightarrow 0^+} f'(x) = -4(1) = -4 \checkmark \checkmark \checkmark \quad \checkmark \checkmark \text{method}$$

Therefore differentiable at $x = 1$. $\checkmark$ (9)

14 marks

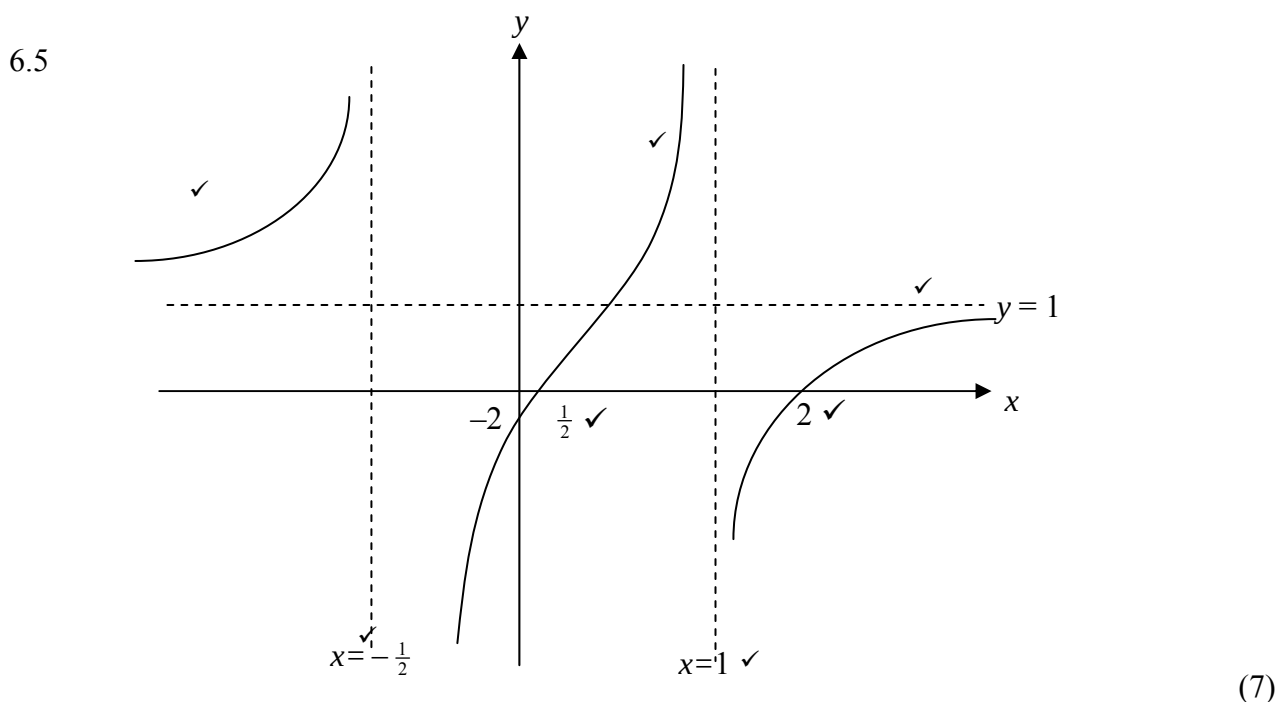
QUESTION 6

6.1 $g(x) = \frac{2x^2 - 5x + 2}{2x^2 - x - 1}$ or $2x^2 - 5x + 2 = 0$
 $= \frac{(2x-1)(x-2)}{(2x+1)(x-1)}$ ✓ $(2x-1)(x-2) = 0$ ✓
 $x = \frac{1}{2}$ or $x = 2$ ✓ (2)

6.2 $\frac{2x^2 - x - 1}{(2x+1)(x-1)}$ ✓
 Vertical: $x = -\frac{1}{2}$ and $x = 1$ ✓✓
 $\lim_{x \rightarrow \infty} \frac{2 - \frac{5}{x} + \frac{2}{x^2}}{2 - \frac{1}{x} - \frac{1}{x^2}} \checkmark \checkmark \checkmark = 1$
 Horizontal: $y = 1$ ✓ (7)

6.3 $g'(x) = \frac{(2x^2 - x - 1)(4x - 5) - (2x^2 - 5x + 2)(4x - 1)}{(2x^2 - x - 1)^2}$ ✓✓✓✓✓✓
 $= \frac{8x^2 - 12x + 7}{(2x^2 - x - 1)^2}$ ✓✓ (8)

6.4 $\Delta = 144 - 4(8)(7) = -80$ ✓
 $8x^2 - 12x + 7 \neq 0$ ✓
 Therefore no local max or min turning point. ✓ (3)



27 marks

QUESTION 7

$$7.1 \quad f'(x) = \frac{1}{2}(x + \sqrt{x})^{-\frac{1}{2}} \left(1 + \frac{1}{2}x^{-\frac{1}{2}}\right) \checkmark \checkmark \checkmark \checkmark \checkmark \checkmark \quad (6)$$

$$7.2 \quad \begin{aligned} &\cos(a-x)\cos x + \sin x(-\sin(a-x)(-1)) \checkmark \checkmark \checkmark \checkmark \\ &= \cos(a-x)\cos x + \sin(a-x)\sin x \checkmark \\ &= \cos(a-x-x) \checkmark \\ &= \cos(a-2x) \checkmark \end{aligned} \quad (7)$$

13 marks**QUESTION 8**

$$8.1 \quad \begin{aligned} \frac{dy}{dt} &= 2t \checkmark & \frac{dx}{dt} &= 3 \checkmark \\ \frac{dy}{dx} &= 2t \times \frac{1}{3} \checkmark \checkmark & &= \frac{2}{3}t \end{aligned} \quad (4)$$

$$8.2 \quad \begin{aligned} 2 &= 2t \times \frac{1}{3} \checkmark \checkmark \\ t &= 3 \checkmark \\ x &= 3(3) + 1 = 10 \checkmark \\ y &= 3^2 = 9 \quad \checkmark \\ (10;9) & \end{aligned} \quad (5)$$

9 marks**QUESTION 9**

$$9.1 \quad 3 \text{ solutions} \checkmark \checkmark \quad (2)$$

$$9.2 \quad x = 0,5 \checkmark \quad (1)$$

$$9.3 \quad \begin{aligned} -x + 1 &= x^3 - 5x^2 + 6x \quad \checkmark \checkmark \checkmark \\ \therefore 0 &= x^3 - 5x^2 + 7x - 1 \end{aligned}$$

$$x_{r+1} = x_r - \frac{x^3 - 5x^2 + 7x - 1}{3x^2 - 10x + 7} \quad \checkmark \checkmark \checkmark \checkmark$$

$$x_1 = 0.5 \checkmark$$

$$x_2 = 0$$

$$x_3 = \dots$$

$$\dots\dots\dots$$

$$x = 0,160713 \text{ (6 dp)} \quad \checkmark \checkmark \quad (10)$$

13 marks

QUESTION 10

$$10.1 \quad y' = 3ax^2 + 2bx + c \quad \checkmark\checkmark\checkmark$$

$$y'' = 6ax + 2b$$

$$0 = 6a(2) + 2b \quad \checkmark\checkmark$$

$$b = -6a$$

(5)

$$10.2 \quad -3 = 3a(0) + 2b(0) + c \quad \checkmark\checkmark\checkmark$$

$$\therefore c = -3$$

Passes through (0;0) $\therefore d = 0 \quad \checkmark$

Passes through (2; -22)

$$\therefore -22 = 8a + 4b - 6 \quad \checkmark\checkmark$$

$$\therefore b = -2a - 4$$

$$\therefore -6a = -2a - 4$$

$$\therefore a = 1 \quad \checkmark$$

$$\therefore b = -6 \quad \checkmark$$

$$y = x^3 - 6x^2 - 3x \quad \checkmark$$

(9)

14 marks**QUESTION 11**

$$11.1 \quad P = 3 \times \left(2 \times \frac{\pi}{3} \right) \quad \checkmark\checkmark\checkmark$$

$$= 6,28 \text{ cm} \quad \checkmark$$

(3)

$$11.2 \quad A = \Delta ABC + 3 \text{ segments} = \frac{1}{2} \times 2 \times 2 \times \sin \frac{\pi}{3} + 3 \left[\frac{1}{2} \times 2^2 \times \frac{\pi}{3} - \frac{1}{2} \times 2 \times 2 \times \sin \frac{\pi}{3} \right] \quad \checkmark\checkmark\checkmark\checkmark\checkmark\checkmark\checkmark\checkmark$$

$$= 2,82 \text{ cm}^2 \quad \checkmark$$

or $A = 1 \text{ Sector} + 2 \text{ segments}$

$$= \frac{1}{2} \times 2^2 \times \frac{\pi}{3} + 2 \left[\frac{1}{2} \times 2^2 \times \frac{\pi}{3} - \frac{1}{2} \times 2 \times 2 \times \sin \frac{\pi}{3} \right]$$

or $A = 3 \text{ sectors} - 2 \text{ triangles}$

$$= 3 \left[\frac{1}{2} \times 2^2 \times \frac{\pi}{3} \right] - 2 \left[\frac{1}{2} \times 2 \times 2 \times \sin \frac{\pi}{3} \right]$$

(8)

11 marks

QUESTION 12

$$\begin{aligned}
 12.1 \quad \int_2^3 x(x^2 - 1)^{-\frac{1}{2}} dx & \checkmark & \text{or} & \quad \text{Let} \quad u = x^2 - 1 \\
 & = \frac{1}{2} \int_2^3 2x(x^2 - 1)^{-\frac{1}{2}} dx & \checkmark \checkmark & \quad \frac{du}{dx} = 2x \\
 & = \left[(x^2 - 1)^{\frac{1}{2}} \right]_2^3 & \checkmark \checkmark \checkmark \checkmark & \quad dx = \frac{du}{2x} \\
 & = \sqrt{8} - \sqrt{3} = 1,10 \text{ units} & \checkmark \checkmark \checkmark & \quad \int_2^3 \frac{x}{\sqrt{x^2 - 1}} dx = \int_3^8 \frac{1}{2\sqrt{u}} du \\
 & & & \quad = \left[u^{\frac{1}{2}} \right]_3^8 \\
 & & & \quad = \sqrt{8} - \sqrt{3} \\
 & & & \quad = 1,10 \quad (10)
 \end{aligned}$$

$$\begin{aligned}
 12.2 \quad (a) \quad \frac{5-3}{5} &= 0,4 \checkmark \checkmark \\
 x_1 &= 3,4 & h_1 &= \ln 3,4 & Area_1 &= 0,4895 \\
 x_2 &= 3,8 & h_2 &= \ln 3,8 & Area_2 &= 0,5340 \\
 x_3 &= 4,2 & h_3 &= \ln 4,2 & Area_3 &= 0,5740 \checkmark \checkmark \checkmark \checkmark \checkmark \checkmark \checkmark \\
 x_4 &= 4,6 & h_4 &= \ln 4,6 & Area_4 &= 0,6104 \\
 x_5 &= 5 & h_5 &= \ln 5 & Area_5 &= 0,6438 \\
 \text{TOTAL} &= 2,852 \text{ units}^2 \checkmark & & & & (10)
 \end{aligned}$$

$$\begin{aligned}
 (b) \quad [x \ln x - x]_3^5 & \checkmark \\
 &= 5 \ln 5 - 5 - 3 \ln 3 + 3 \checkmark \checkmark \checkmark \checkmark \\
 &= 2,751 \text{ units}^2 \checkmark \\
 & & & (6)
 \end{aligned}$$

26 marks

QUESTION 13

$$\begin{aligned}
 V &= \int_{-1}^1 \pi(4 - 4x^2) dx - \int_{-1}^1 \pi(1 - x^2) dx \checkmark \checkmark \checkmark \checkmark & V &= \int_{-1}^1 \pi(3 - 3x^2) dx \\
 &= \pi \left[4x - \frac{4x^3}{3} \right]_{-1}^1 - \pi \left[x - \frac{x^3}{3} \right]_{-1}^1 \checkmark \checkmark \checkmark \checkmark & &= \pi [3x - x^3]_{-1}^1 \\
 &= \pi \left[4 - \frac{4}{3} + 4 - \frac{4}{3} \right] - \pi \left[1 - \frac{1}{3} + 1 - \frac{1}{3} \right] \checkmark \checkmark \checkmark \checkmark & &= \pi [(3-1) - (-3+1)] \\
 &= 4\pi \text{ units}^3 \checkmark & &= 4\pi \text{ units}^3
 \end{aligned}$$

13 marks

MODULE 2 STATISTICS**QUESTION 1**

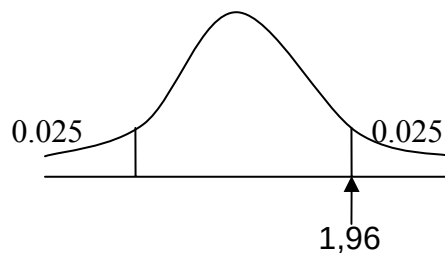
1.1 $H_0 = 3118$ ✓✓✓✓ (4)
 $H_1 \neq 3118$

- 1.2 Testing for change therefore involves two rejection regions. 5% level of significance means each region is 2,5%. ✓ The critical value that corresponds is 1,96. ✓

$$Z = \frac{x - \mu}{\frac{\sigma}{\sqrt{n}}} \checkmark$$

$$Z = \frac{3345 - 3118}{\frac{850,5}{\sqrt{40}}} \checkmark \checkmark \checkmark$$

$$Z = 1,688$$



This value of Z falls within the acceptance region, therefore not enough evidence to reject H_0 . Accept H_0 ✓ (7)

1.3 $1,96 = \frac{x - 3118}{\frac{850,5}{\sqrt{40}}} \checkmark \checkmark$

$x = 3381,6g \checkmark \checkmark$ (4)

15 marks

QUESTION 2

2.1 As x increases y decreases. ✓✓ (2)

2.2 (a) $r = -0,98$ ✓✓✓✓ (4)

(b) very strong correlation ✓ (1)

$$2.3 \quad b = \frac{n \sum xy - \sum x \sum y}{n \sum x^2 - (\sum x)^2} \quad \checkmark \checkmark$$

$$n = 12 \quad \checkmark$$

$$b = \frac{12(3291,88) - 511(78,52)}{12(289490) - (511)^2} \quad \checkmark \checkmark \checkmark$$

$$b = -0,0072$$

$$y - \bar{y} = b(x - \bar{x})$$

Now for a: $y - 6,54 = -0,0072(x - 42,58)$ ✓✓✓✓ (10)

$$y = -0,0072x + 6,85$$

2.4 (a) $y = 6,85 - 0,0072(95)$ ✓✓✓ (3)

$$y = 6,17$$

(b) unreliable, outside of range, extrapolation ✓✓ (2)

22 marks

QUESTION 3

3.1 Proportion not in favour = $\frac{202}{700}$ ✓

Proportion in favour = $\frac{498}{700}$

90% confidence interval $z = 1,645$ ✓

$$p \pm 1,645 \sqrt{\frac{p(1-p)}{n}} \quad \checkmark$$

$$\frac{202}{700} \pm 1,645 \sqrt{\frac{\frac{202}{700} \times \frac{498}{700}}{700}} \quad \checkmark \checkmark \checkmark$$

$$[0,260399; 0,316742] \quad \checkmark \checkmark \checkmark \checkmark$$

(10)

3.2 (a) $\frac{105,89 + 110,11}{2} = 108$ ✓ ✓

(2)

(b) $105,89 = 108 - 1,96 \frac{\sigma}{\sqrt{700}}$ ✓ ✓ ✓ ✓

(6)

$$\sigma = 28,48 \quad \checkmark \checkmark$$

3.3 $1 - \binom{10}{1} (0,55)^1 (0,45)^9 - \binom{10}{0} (0,45)^{10}$ ✓ ✓ ✓ ✓

$$= 0,995 \quad \checkmark \checkmark$$

(9)

27 marks

QUESTION 4

4.1 $8! - 7! \times 2! \checkmark\checkmark$
 $= 30240 \checkmark$ (3)

4.2 (a) $\frac{0,1}{0,5} = \frac{1}{5} \checkmark$ (1)

(b) $\frac{0,4}{0,5} = \frac{4}{5} \checkmark$ (1)

(c) $P(A) \cdot P(B) = 0,4 \times 0,5$
 $= 0,2 \checkmark$
 and $P(A \cap B) = 0,1 \checkmark$
 $\therefore$ not equal
 $\therefore$ not independent $\checkmark$ (3)

4.3 (a) $P(X = x) = \begin{cases} \frac{\binom{4}{x} \binom{8}{3-x}}{\binom{12}{3}} & \text{for } x = 0,1,2,3 \quad \text{formula } \checkmark\checkmark\checkmark\checkmark\checkmark \\ 0 & \text{elsewhere} \quad \checkmark \end{cases}$ (6)

(b) $\frac{\binom{4}{2} \binom{8}{1}}{\binom{12}{3}} = \frac{12}{55} \checkmark\checkmark\checkmark\checkmark\checkmark$ (5)

4.4 (a) $\int_2^6 (0,2 - 0,02x) dx \checkmark\checkmark$
 $\left. \frac{x}{5} - \frac{x^2}{100} \right|_2^6 \checkmark\checkmark$
 $= \frac{6}{5} - \frac{36}{100} - \left(\frac{2}{5} - \frac{4}{100} \right) \checkmark\checkmark$
 $= 0,48 \checkmark\checkmark$
 Probability delay would be between 2 and 6 hours is 0,48 (8)

(b) $\left. \frac{x}{5} - \frac{x^2}{100} \right|_0^m \checkmark\checkmark = 0,5 \checkmark\checkmark$
 $\frac{m}{5} - \frac{m^2}{100} = 0,5 \checkmark$
 $20m^2 - 400m + 1000 = 0 \checkmark\checkmark\checkmark\checkmark$
 $m = 2,93 \quad \text{or} \quad m = 17,07 (N/A)$ (9)

36 marks

MODULE 3 FINANCE AND MODELLING**QUESTION 1**

1.1

$$\begin{aligned}
 & \checkmark \quad \checkmark \\
 & P(1-0,2)^5(1-0,4)^2 = P(1-i)^7 \quad \checkmark \\
 & 0,1179648 = (1-i)^7 \quad \checkmark \\
 & 0,7368... = 1-i \quad \checkmark \\
 & i = 26,31\% \quad \checkmark
 \end{aligned}$$

(6)

- 1.2 A Paying more than the required amount in order to pay back more quickly. ✓✓
 B Cash flow problem - Paying only sufficient to cover interest. ✓✓
 C Interest rates rise. Paying less than the required amount. ✓✓
 D Gets promotion, bigger salary. Able to pay more per month and finish the loan in the right time. ✓✓

(8)

14 marks**QUESTION 2**

$$2.1 \quad 6000000 \left(1 + \frac{0,095}{12}\right)^3 = 55000 \left[\frac{1 - \left(1 + \frac{0,095}{12}\right)^{-n}}{\frac{0,095}{12}} \right] \quad \checkmark \checkmark$$

$$n = 273,5 \text{ months } \checkmark \checkmark \checkmark$$

$$\text{So } 276 \text{ months in total} = 23 \text{ years } \checkmark$$

(10)

2.2 Amount remaining =

$$6000000 \left(1 + \frac{0,095}{12}\right)^{24} - 55000 \left[\frac{\left(1 + \frac{0,095}{12}\right)^{21} - 1}{\frac{0,095}{12}} \right] \quad \checkmark \checkmark$$

$$= R7\,250\,071,51 \quad \checkmark \checkmark - R1\,251\,189,89 \quad \checkmark \checkmark$$

$$= 5\,998\,881,62 \quad \checkmark$$

(12)

22 marks

QUESTION 3

$$3.1 \quad 25000 \left(1 + \frac{0,16}{12}\right)^{60} + 25000 \left(1 + \frac{0,16}{12}\right)^{59} + 25000 \left(1 + \frac{0,16}{12}\right)^{58} = R163860,42 \quad (8)$$

$$3.2 \quad 163860,42 = x \left[\frac{\left(1 + \frac{0,16}{12}\right)^{58} - 1}{\frac{0,16}{12}} \right] \quad x = R1890,08 \quad (9)$$

17 marks**QUESTION 4**

$$4.1 \quad 132 = 30q - 3(6) \quad q = 5$$

$$30 = 5(6) - 2p \quad p = 0 \quad (8)$$

$$4.2 \quad T_{n+1} = 100 \times 1,1^n + \left(1 + \frac{0,14}{12}\right) T_n$$

15 marks

QUESTION 5

- 5.1 (a) Rabbits = 30, ✓ Foxes = 5 ✓ (2)
- (b) No. ✓ The number of rabbits and foxes are tending to a limit. ✓✓✓ (4)
- (c) Rabbits = 55. ✓ Foxes = 8 ✓ (2)
- (d) $\checkmark$ $\checkmark$ $\checkmark$ $\checkmark$
 $30 < r < 55$ $4 < f < 8$ (4)
- 5.2 (a) In the pred-prey model one must include the influence of the contact ✓ between rabbits and foxes. The term represents the number of rabbit deaths ✓ and is a function of the product of the number of rabbits and foxes. ✓ (3)
- (b) $R = \frac{c}{fb}$ ✓✓
 $= \frac{0,048}{0,12 \times 0,008}$ ✓
 $= 50$ ✓
 $F = \frac{a}{b} \left(1 - \frac{c}{fbK} \right)$ ✓✓
 $= \frac{0,64}{0,008} \left(1 - \frac{0,048}{0,12 \times 0,008 \times 400} \right)$ ✓
 $= 70$ ✓ (8)

23 marks

QUESTION 6

- 6.1 A Logistic ✓ model since there is a limit ✓ to the number of ants over time. (2)
- 6.2 (a) 2 500 ✓✓ (2)
- (b) $375 = 250 + 250r \left(1 - \frac{250}{2500} \right)$ ✓✓✓
 $0,5 = \frac{9r}{10}$ ✓
 $r = \frac{5}{9}$ ✓ (5)

9 marks

MODULE 4 MATRICES AND GRAPHS**QUESTION 1**

1.1 (a) Enlargement scale factor 2 $\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$ (4)

(b) Shear factor 1, invariant y-axis: $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ (5)

1.2 (a) Rotation -90° and stretch factor 2, parallel to x-axis. (5)

(b) $\begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 2 \\ -1 & 0 \end{pmatrix}$ (6)

1.3 $\begin{pmatrix} \cos 60 & -\sin 60 \\ \sin 60 & \cos 60 \end{pmatrix} \begin{pmatrix} 10 \\ 6 \end{pmatrix} = \begin{pmatrix} 5 - 3\sqrt{3} \\ 3 + 5\sqrt{3} \end{pmatrix}$ (6)

26 marks**QUESTION 2**

2.1 *cofactors*: $\begin{pmatrix} 9 & 6 & 4 \\ 4 & -12 & -8 \\ -3 & -2 & -16 \end{pmatrix}$ ✓✓✓✓

adj = $\begin{pmatrix} 9 & 4 & -3 \\ 6 & -12 & -2 \\ 4 & -8 & -16 \end{pmatrix}$ ✓

$\begin{pmatrix} 9 & 4 & -3 \\ 6 & -12 & -2 \\ 4 & -8 & -16 \end{pmatrix} \begin{pmatrix} 4 & 2 & -1 \\ 2 & -3 & 0 \\ 0 & 2 & -3 \end{pmatrix} = \begin{pmatrix} 44 & 0 & 0 \\ 0 & 44 & 0 \\ 0 & 0 & 44 \end{pmatrix}$ ✓✓✓✓

inverse = $\frac{1}{44} \begin{pmatrix} 9 & 4 & -3 \\ 6 & -12 & -2 \\ 4 & -8 & -16 \end{pmatrix}$ ✓

(10)

2.2 $\frac{1}{44} \begin{pmatrix} 9 & 4 & -3 \\ 6 & -12 & -2 \\ 4 & -8 & -16 \end{pmatrix} \begin{pmatrix} 3 \\ 4 \\ 7 \end{pmatrix} = \begin{pmatrix} 0,5 \\ -1 \\ -3 \end{pmatrix}$ (6)

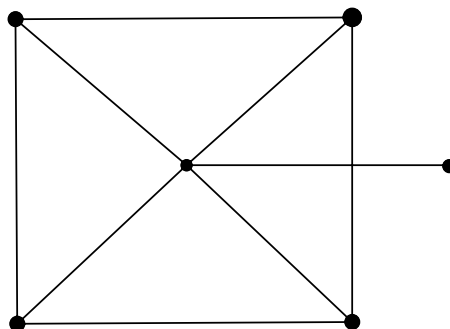
16 marks

QUESTION 3

- (1) Two planes parallel. ✓✓✓
 (2) $\det = -3$, so unique solution ✓✓✓
 (3) $\det = 0$, planes not parallel, so this is the one. ✓✓✓✓

10 marks**QUESTION 4**

- 4.1 18, since 9 edges ✓✓ (2)
 4.2 Both the vertices of order five must go to all other vertices ✓✓ hence the minimum order of any other vertex must be 2 ✓ (3)
 4.3 One with order 5 ✓✓ four with an order of 3 ✓✓ one with an order of 1 ✓



(5)

10 marks**QUESTION 5**

- 5.1 B, D, H, I ✓✓ (2)
 5.2 (Chinese Postman Problem)
 – Recognises the need to travel a route twice ✓
 – (Routes BED and IH repeated is not a minimum route)
 – Routes BEI ✓✓ and DGH ✓✓ are repeated
 – A solution that starts and finishes at A ✓✓
 – Every edge has been travelled at least once ✓✓
 – Solution for the shortest inspection route:
 e.g. $ABCDEBE D G F H G H I E I G C A$ ✓✓✓ (14)
 5.3 Summing all the edges ✓
 $+ BEI \text{ repeat} + DGH \text{ repeat} = 51,4 + 5,6 \checkmark + 4,3 \checkmark = 61,3 \text{ km } \checkmark$ (4)
 5.4 Yes ✓✓ BD would remove two of the vertices with odd order ✓ and $HI = 3,4 \checkmark \checkmark$
 Total travelled would be shorter than before at $51,4 + 6,4 + 3,4 = 61,2 \text{ km } \checkmark$ (6)

26 marks

QUESTION 6

- 6.1 Students must demonstrate that they have used Prim's algorithm, rather than just an intuitive approach or Kruskal's algorithm – students lose 50% of the mark if they do not demonstrate use of the correct algorithm ✓✓

1. E2 → Office (488 m) ✓✓
2. E2 → E4 (587 m) ✓✓
3. E4 → E3 (523 m) ✓✓
4. Office → E1 (1514 m) ✓✓ (10)

- 6.2 $488 + 587 + 523 + 1514 = 3112 \text{ m}$ ✓✓ (2)

12 marks

Total: 300 marks