



GRAAD 12-EKSAMEN
NOVEMBER 2011

GEVORDERDEPROGRAM-WISKUNDE

NASIENRIGLYNE

Tyd: 3 uur

300 punte

Hierdie nasienriglyne is as basis tydens die amptelike IEB nasiensessie gebruik. Hierdie nasienriglyne is voorberei vir gebruik deur eksaminatore en hulpeksaminatore. Daar is van alle nasieners vereis om 'n standaardiseringsvergadering by te woon om te verseker dat die nasienriglyne konsekwent vertolk en regverdig toegepas word tydens die bepunting van kandidate se skrifte.

Tydens die standaardiseringsvergaderings word besluite geneem rakende die toekenning van punte in die belang van regverdigheid teenoor alle kandidate in die konteks van 'n algehele summatiewe assessering.

Die IEB sal geen gesprek aanknoop of korrespondensie voer oor enige nasienriglyne nie. Daar word toegegee dat verskillende menings rondom sake van beklemtoning of detail in sodanige riglyne mag voorkom. Gevolglik word die spesifieke puntetoekenning nie aangedui nie.

MODULE 1 CALCULUS EN ALGEBRA

VRAAG 1

$$1.1 \quad p = 3\ln 3 \\ = \ln 27$$

$$q = 3\ln 6 \\ = \ln 216$$

$$\begin{aligned} \text{Gradiënt} &= \frac{q - p}{6 - 3} \\ &= \frac{(\ln 216) - (\ln 27)}{3} \\ &= \frac{3\ln 6 - 3\ln 3}{3} \\ &= \ln 2 \end{aligned}$$

OF

$$\begin{aligned} q - p &= 3\ln 6 - 3\ln 3 \\ &= 3\ln 6 - 3\ln 3 \\ &= 3\ln 2 \end{aligned}$$

$$\begin{aligned} \text{Gradiënt} &= \frac{3\ln 2}{6 - 3} \\ &= \ln 2 \end{aligned}$$

(7)

$$1.2 \quad f(n+1) - 8f(n) = p \times 15^n$$

$$\therefore [15^{n+1} - 8^{n+1-2}] - 8[15^n - 8^{n-2}] = p \times 15^n$$

$$\therefore 15^{n+1} - 8^{n-1} - 8 \cdot 15^n + 8^{n-1} = p \times 15^n$$

$$\therefore 15^n(15 - 8) = p \times 15^n$$

$$\therefore 15^n(7) = p \times 15^n$$

$$\therefore p = 7$$

(8)

[15]

VRAAG 2

$$2.1 \quad (a) \quad \text{Neem aan (stelling) waar vir } n = k \quad (1)$$

$$\begin{aligned} (b) \quad \text{LK} &= [1 + 8 + 27 + \dots + k^3] + [(k+1)^3] \quad \text{OF} \\ &= \frac{k^2(k+1)^2}{4} + (k+1)^3 \text{ uit (5)} \\ &= \frac{k^2(k+1)^2 + 4(k+1)^3}{4} \\ &= \frac{(k+1)^2 [k^2 + 4(k+1)]}{4} \\ &= \frac{(k+1)^2 [k^2 + 4k + 4]}{4} \\ &= \frac{(k+1)^2 (k+2)^2}{4} \\ &= \text{RK} \end{aligned}$$

$$\begin{aligned} &= \frac{k^2(k^2 + 2k + 1) + 4(k^3 + 3k^2 + 3k + 1)}{4} \\ &= \frac{k^4 + 2k^3 + k^2 + 4k^3 + 12k^2 + 12k + 4}{4} \\ &= \frac{k^4 + 6k^3 + 13k^2 + 12k + 4}{4} \\ \text{RK} &= \frac{(k^2 + 2k + 1)(k^2 + 4k + 4)}{4} \\ &= \frac{k^4 + 6k^3 + 13k^2 + 12k + 4}{4} \\ \therefore \text{LK} &= \text{RK} \end{aligned}$$

(8)

$$2.2 \quad \Delta x = \frac{2-0}{n} = \frac{2}{n}$$

$$x_i = \frac{2i}{n}$$

$$f(x_i) \Delta x = \left(\frac{2i}{n} \right)^3 + 1 = \frac{8i^3}{n^3} + 1$$

$$\text{Oppervlakte} = \lim_{n \rightarrow \infty} \frac{b-a}{n} \sum_{i=1}^n f(x_i)$$

$$\lim_{n \rightarrow \infty} \sum_{i=1}^n \left(\frac{8i^3}{n^3} + 1 \right)$$

$$= \lim_{n \rightarrow \infty} \frac{8}{n^3} \left[\frac{n^2(n^2 + 2n + 1)}{4} \right] + n$$

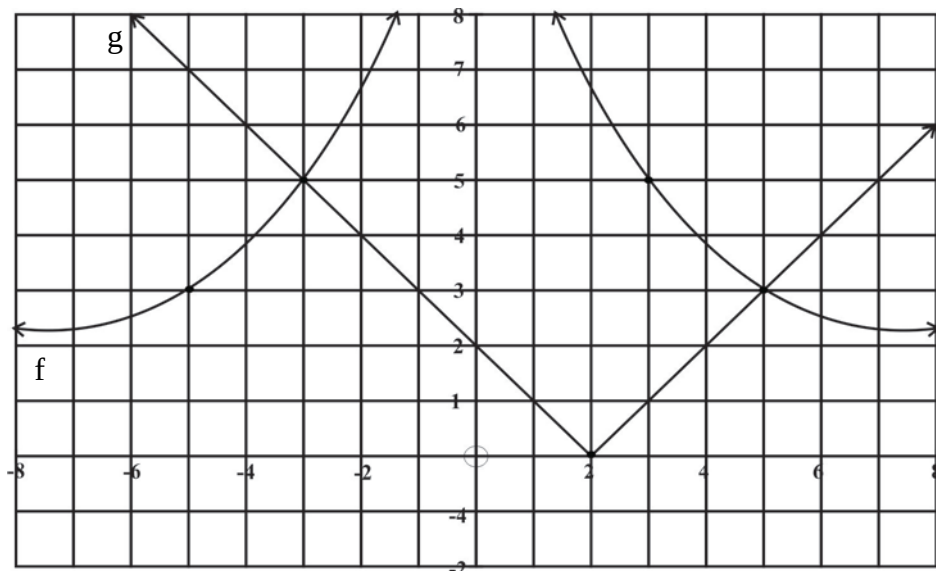
$$= \lim_{n \rightarrow \infty} 2n + 4 + \frac{2}{n} + n$$

$$\begin{aligned} \text{Oppervlakte} &= \lim_{n \rightarrow \infty} \frac{2}{n} \left(3n + 4 + \frac{2}{n} \right) \\ &= 6 \end{aligned}$$

(11)
[20]

VRAAG 3

3.1



(7)

$$\begin{aligned}
 3.2 \quad & x \in [-3; 0) \cup (0; 5] \\
 & \text{of: } -3 \leq x < 0 \quad \text{of} \quad 0 < x \leq 5 \\
 & \text{of: } -3 \leq x \leq 5; \quad x \neq 0
 \end{aligned}$$

(4)
[11]

VRAAG 4

$$\begin{aligned}
 4.1 \quad & \text{Oppervlakte segment AOB} = \frac{1}{2} r^2 \theta \quad \text{OF} \quad \text{Oppervlakte segment AOB} = \frac{1}{2} r^2 \theta \\
 & \text{Oppervlakte } \triangle AOX = \frac{1}{2} AO \cdot OX \sin \theta \quad \text{Oppervlakte } \triangle AOX = \frac{1}{2} OX \cdot XA \\
 & OX = r \cos \theta \quad XA = r \sin \theta \\
 & \therefore \triangle AOX = \frac{1}{2} r \cdot r \cos \theta \sin \theta \quad \text{Oppervlakte } \triangle AOX = \frac{1}{2} r \cos \theta \cdot r \sin \theta \\
 & \quad = \frac{1}{2} r^2 \cos \theta \sin \theta \quad = \frac{1}{2} r^2 \cos \theta \sin \theta \\
 & \text{Oppervlakte AXB} = \frac{1}{2} r^2 \theta - \frac{1}{2} r^2 \cos \theta \sin \theta \quad \therefore \text{Oppervlakte AXB} = \frac{1}{2} r^2 \theta - \frac{1}{2} r^2 \cos \theta \sin \theta \\
 & \quad = \frac{1}{2} r^2 (\theta - \sin \theta \cos \theta) \text{ cm}^2 \quad = \frac{1}{2} r^2 (\theta - \sin \theta \cos \theta) \text{ cm}^2 \\
 & \quad \quad \quad (7)
 \end{aligned}$$

4.2 Omtrek AXB

$$\begin{aligned}
 & = AX + AB + XB \\
 & = r \sin \theta + r\theta + (r - r \cos \theta) \\
 & = 12 \sin \frac{\pi}{6} + 12 \left(\frac{\pi}{6} \right) + 12 - 12 \cos \frac{\pi}{6} \quad (\text{vervang in}) \\
 & = 6 + 2\pi + 12 - 6\sqrt{3} \\
 & = 18 + 2\pi - 6\sqrt{3} \text{ cm}
 \end{aligned}$$

(8)
[15]

VRAAG 5

$$\begin{aligned}
 5.1 \quad R_1 &= -\frac{1}{2} + \frac{\sqrt{3}}{2}i & ; R_3 &= -1 + 2\sqrt{3} \\
 \therefore R_2 &= -\frac{1}{2} - \frac{\sqrt{3}}{2}i & \therefore R_4 &= -1 - 2\sqrt{3} \\
 (SR)_1 &= -1 & (SR)_2 &= -2 \\
 (PR)_1 &= \frac{1}{4} - \frac{3}{4}i^2 & (PR)_2 &= 1 - 12 \\
 &= \frac{1}{4} + \frac{3}{4} & &= -11 \\
 &= 1 & &
 \end{aligned}$$

$$\begin{aligned}
 \therefore 2 \text{ kwadratiese faktore} &= (x^2 - SR, x + PR_1)(x^2 - SR_2x + PR_2) \\
 &= (x^2 + x + 1)(x^2 + 2x - 11) \\
 \therefore P(x) &= (x^2 + x + 1)(x^2 + 2x - 11)(x + 2) \quad (14)
 \end{aligned}$$

OF

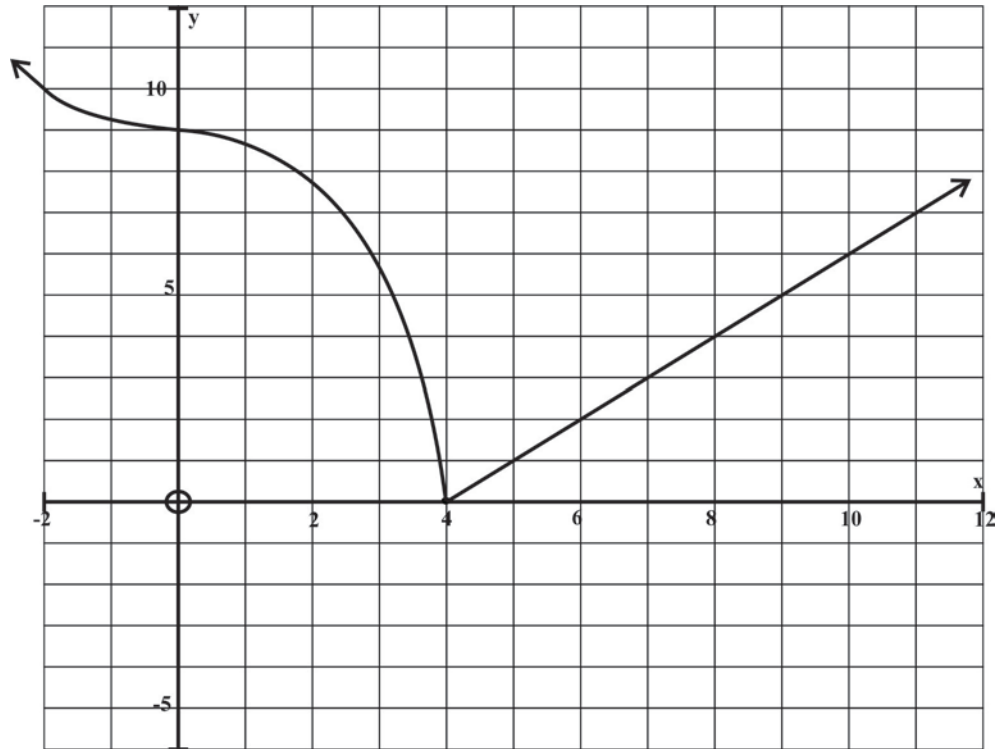
$$\begin{aligned}
 \left(x + \frac{1}{2}\right)^2 &= \left(\frac{\pm\sqrt{3}}{2}i\right)^2 \\
 x^2 + x + \frac{1}{4} &= \frac{3}{4}i^2 \\
 x^2 + x + 1 &= 0 \text{ en} \\
 (x + 1)^2 &= (\pm 2\sqrt{3})^2 \\
 x^2 + 2x + 1 &= 12 \\
 x^2 - 2x - 11 &= 0 \\
 \therefore P(x) &= (x^2 + x + 1)(x^2 + 2x - 11)(x + 2)
 \end{aligned}$$

$$\begin{aligned}
 5.2 \quad \text{Reële wortels: } x &= -1 \pm 2\sqrt{3}; x = -2 & (2) \\
 & & [16]
 \end{aligned}$$

VRAAG 6

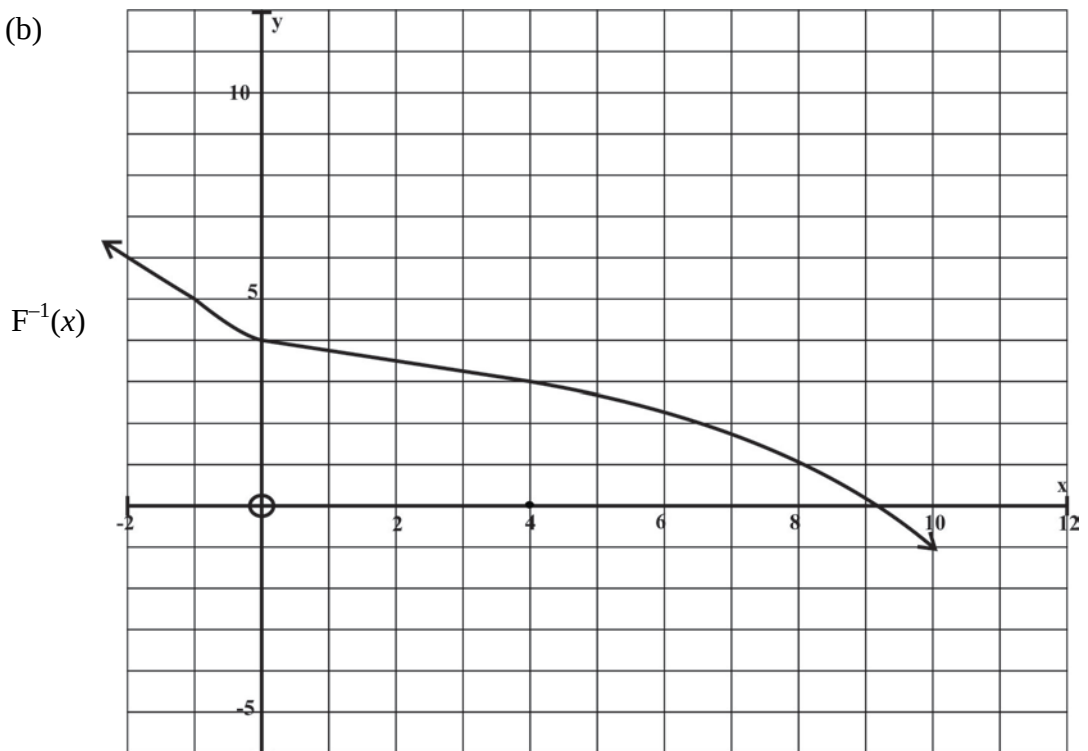
- 6.1
- (a) $x = 3$ (2)
 - (b) $x = -1$ (2)
 - (c) $x = 1$ (2)
 - (d) $x = -2; x > 3$ (3)
 - (e) $x < -1$ (grafiek konkaaf op) (3)

6.2 (a)



(3)

(b)



(5)
[20]

VRAAG 7

$$7.1 \quad y = x^{\frac{1}{3}} \cos(x^2 - 1)$$

$$\frac{dy}{dx} = \frac{1}{3} x^{-\frac{2}{3}} \cdot \cos(x^2 - 1) + x^{\frac{1}{3}} \sin(x^2 - 1) \cdot 2x$$

(9)

$$7.2 \quad (a) \quad f(x) = \cot x$$

$$f'(x) = -\operatorname{cosec}^2 x \quad (1)$$

$$f''(x) = (-2\operatorname{cosec} x)(-\cot x \operatorname{cosec} x) \quad (3)$$

$$(b) \quad f'\left(\frac{\pi}{2}\right) = -\left(\operatorname{cosec} 2 \frac{\pi}{2}\right) = -1$$

$$\therefore f'\left(\frac{\pi}{2}\right) \neq 0 \quad (2)$$

$$\begin{aligned} f''\left(\frac{\pi}{2}\right) &= \left(-2\operatorname{cosec} - \frac{\pi}{2}\right) \left(\pi \cot \frac{\pi}{2} \cdot \operatorname{cosec} \frac{\pi}{2}\right) \\ &= -2(1)(0) \\ &= 0 \end{aligned} \quad (3)$$

$$\therefore \text{Niestasionêre buigpunt wanneer } x = \frac{\pi}{2}$$

OF

$$7.3 \quad 2 + 3y^1 - (y^2 + 2xy y^1) = 0$$

$$y^1(3 - 2xy) = y^2 - 2$$

$$y^1 = \frac{y^2 - 2}{(3 - 2xy)}$$

$$\begin{aligned} y^1(-1; -2) &= \frac{(-2)^2 - 2}{3 - 2(-1)(-2)} \\ &= \frac{2}{3 - 4} \\ &= -2 \end{aligned}$$

$$\therefore y - y_1 = m(x - x_1)$$

$$\therefore y + 2 = -2(x + 1)$$

$$\therefore y = -2x - 4$$

$$2 + 3\frac{dy}{dx} - y^2 - 2xy \cdot \frac{dy}{dx} = 0$$

$$\frac{dy}{dx}(3 - 2xy) = y^2 - 2$$

$$\frac{dy}{dx} = \frac{y^2 - 2}{3 - 2xy}$$

$$\therefore \text{wanneer } x = -1; y = -2$$

$$\frac{dy}{dx} = \frac{(-2)^2 - 2}{3 - 2(-1)(-2)}$$

$$= \frac{2}{3 - 4}$$

$$= -2$$

$$\text{of } y = mx + c$$

$$-2 = -2(-1) + c$$

$$c = -4$$

$$\therefore y = -2x - 4 \quad (13)$$

[31]

VRAAG 8

$$8.1 \quad (a) \quad f(x) = \frac{3(x-3)}{(x-2)(x+1)}$$

Vertikale asimptoot $x = 2; x = -1$

Horisontale asimptoot $y = 0$

(5)

(b) By A, $f'(x) = 0$

$$\therefore (x^2 - x - 2)(3) - (3x - 9)(2x - 1) = 0$$

$$N = 0; D \neq 0$$

$$\therefore 3x^2 - 3x - 6 - 6x^2 + 21x - 9 = 0$$

$$\therefore 3x^2 + 18x - 15 = 0$$

$$\therefore x^2 - 6x + 5 = 0$$

$$(x - 5)(x - 1) = 0$$

$$x = 5; x = 1$$

NVT

(10)

(c) Aangesien A links is van $x = 3$

(1)

$$8.2 \quad p(x) = \tan\left(x - \frac{\pi}{4}\right) - \frac{x}{2}$$

$$p'(x) = \sec^2\left(x - \frac{\pi}{4}\right) - \frac{1}{2}$$

$$a_{n+1} = a_n - \frac{p(a_n)}{p'(a_n)}$$

$$a_1 = \frac{\pi}{2}$$

$$a_2 = \frac{\pi}{2} - \left[\frac{\tan \frac{\pi}{4} - \frac{\pi}{8}}{\sec^2\left(\frac{\pi}{4}\right) - \frac{1}{2}} \right]$$

$$\tan \frac{\pi}{4} = 1$$

$$= 1,1659$$

$$\sec \frac{\pi}{4} = \sqrt{2}$$

(6)

(OF vir antwoord)

$$\begin{aligned}
 8.3 \quad c(t) &= t(1-t)^{\frac{1}{2}} \\
 c'(t) &= (1-t)^{\frac{1}{2}} + t \cdot \frac{1}{2}(1-t)^{-\frac{1}{2}}(-1) = 0 \\
 2(1-t)^{\frac{1}{2}} &= \frac{t}{(1-t)^{\frac{1}{2}}} \\
 2(1-t) &= t \quad \text{berekening} \\
 2 - 2t &= t \\
 t &= \frac{2}{3} \\
 \text{OF } \therefore \text{ maand } \frac{2}{3}(12) &= \text{Augustus}
 \end{aligned}$$

(9)
[31]

VRAAG 9

$$\begin{aligned}
 9.1 \quad (a) \quad & \text{Laat } u = 5x - 1 \quad \text{OF} \\
 & \begin{cases} du = 5dx \\ \frac{1}{5} du = dx \end{cases} \\
 \therefore \text{Int} &= \frac{1}{5} \int 2(u)^{\frac{1}{2}} du \\
 &= \frac{2}{5} (2) \left(\sqrt{5x-1} \right) + c \quad (5)
 \end{aligned}$$

$$\begin{aligned}
 (b) \quad & u = 5x^2 - 1 \quad \text{OF} \\
 & du = 10x \, du \\
 & \frac{1}{5} du = 2x \, dx \\
 \therefore \text{Int} &= \frac{1}{5} \int u^{-\frac{1}{2}} du \\
 &= \frac{2}{5} (5x^2 - 1)^{\frac{1}{2}} + c \quad (6)
 \end{aligned}$$

$$(c) \quad \int fg^1 dx = fg - \int f^1 g dx$$

$$\text{Laat } f = 2x \quad g^1 = \frac{1}{\sqrt{5x-1}} dx$$

$$f^1 = 2dx$$

$$g = (5x-1)^{-\frac{1}{2}} dx$$

$$\frac{(5x-1)^{-\frac{1}{2}}}{5 \cdot \frac{1}{2}}$$

$$\therefore \text{Int} = \frac{2x \cdot (5x-1)^{-\frac{1}{2}}}{5 \cdot \frac{1}{2}} - 4 \int \frac{(5x-1)^{-\frac{1}{2}}}{5} dx$$

$$\frac{4x(5x-1)^{-\frac{1}{2}}}{5} - \frac{4}{5} \frac{(5x-1)^{\frac{3}{2}}}{\frac{3}{2}} + c \quad (9)$$

$$9.2 \quad (a) \quad \frac{1}{x^2(x+1)} = \frac{a}{x} + \frac{b}{x^2} + \frac{c}{(x+1)}$$

$$\therefore 1 = ax(x+1) + b(x+1) + cx^2$$

OF (deur koëffisiënte gelyk te stel)

$$\text{Wanneer } x = 0: \quad 1 = b$$

$$\text{Wanneer } x = -1 \quad 1 = c$$

$$\text{Wanneer } x = 1 \quad 1 = a(2) + b(2) + c \quad x^2: 0 = a + c$$

$$\therefore 1 = 2a + 2 + 1 \quad \therefore a = -1$$

$$a = -1$$

$$\therefore \frac{1}{x^2(x+1)} = \frac{-1}{x} + \frac{1}{x^2} + \frac{1}{x+1} \quad (6)$$

$$(b) \quad \therefore \int \frac{1}{x^3 + x^2} dx = - \int \frac{1}{x} dx + \int \frac{1}{x^2} dx + \int \frac{1}{x+1} dx$$

$$= \ln|x| + \frac{x^{-1}}{-1} + \ln|x+1| + c$$

$$= \ln\left|\frac{x+1}{x}\right| - x^{-1} + c \quad (5)$$

[31]

VRAAG 10

$$10.1 \quad \text{LK} = \sin^2 x + 2 \sin x \cos x + \cos^2 x$$

$$= 1 + 2 \sin x \cos x$$

(2)

$$10.2 \quad v = \pi \int_0^a y^2 dx$$

$$\therefore \pi \int_0^a (1 + \sin 2x) dx = \pi(a + 1)$$

$$\therefore \left(x - \frac{1}{2} \cos 2x \right) \Big|_0^a = (a + 1)$$

$$\therefore \left(a - \frac{1}{2} \cos 2a \right) - \left(-\frac{1}{2} \cos 0 \right) = a + 1$$

$$\therefore a - \frac{1}{2} \cos 2a + \frac{1}{2} = a + 1$$

$$\therefore \left. \begin{array}{l} -\frac{1}{2} \cos 2a = \frac{1}{2} \\ \cos 2a = -1 \end{array} \right\}$$

$$2a = \pi$$

$$a = \frac{\pi}{2}$$

(8)

[10]

Totaal vir Module 1: 200 punte

MODULE 2

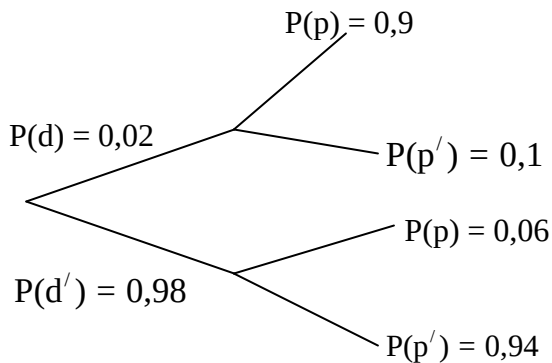
VRAAG 1

$$\binom{10}{8}(0,7)^8(0,3)^2$$

$$= 0,2335$$

[9]

VRAAG 2



$$P(d|p)$$

$$= \frac{P(d \cap p)}{P(p)}$$

$$= \frac{(0,02)(0,9)}{(0,02)(0,9) + (0,98)(0,06)}$$

$$= \frac{15}{64} = 0,2344$$

Alternatief

Aanvaar 'n steekproef van 10 000 mense.

10 000			
Het siekte = 200		Geen siekte = 9 800	
+ Toets	– Toets	+ Toets	– Toets
180	20	588	9 212

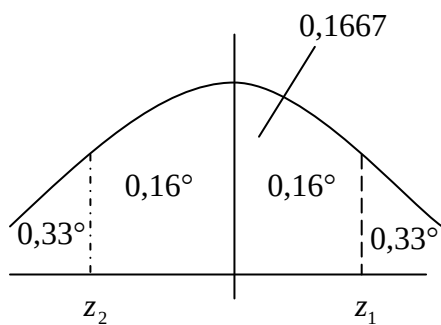
$$\begin{aligned} \text{Getal wat siekte het en + toets} &= 180 \\ \text{Totaal met + toets} &= 180 + 588 \\ \therefore \text{verlangde waarskynlikheid} &= \frac{180}{768} \\ &= 0,2344 \end{aligned}$$

[10]

VRAAG 3

- 3.1 Sterk negatiewe korrelasie. Hoe kouer die temperatuur, hoe meer koppies sjokolade sal verkoop word. (3)
- 3.2 Nee korrelasie tussen die pakkies aartappelskyfies verkoop en die temperatuur. (2)
- 3.3 (a) $y_1 = 413 - 11,22(9)$
 $= 312$ (2)
- (b) $y_2 = 202 + 1,16(21)$
 $= 226$ of korrelasie te laag om 'n voorspelling te maak (2)
- 3.4 y_1 is betroubaar: selfs al word ekstrapolasie gebruik, is die temperatuur naby aan die gegewe databestek en die korrelasiekoëffisiënt is hoog.
 y_2 is nie betroubaar nie: daar is geen korrelasie nie.
 Beide modelle is beperk as die temperatuur te laag of te hoog geëkstrapoleer word. (4)
- [13]**

VRAAG 4



$$\mu = 134$$

$$\sigma = 16$$

$$z_1 = 0,44$$

$$0,43 = \frac{x - 134}{16}$$

$$16(0,43) + 134 = x$$

$$x = 140,88$$

$$z_2 = -0,44$$

$$-0,43 = \frac{x - 134}{16}$$

$$x = 16(-0,44) + 134$$

$$x = 127,12$$

Medium – tussen 127 en 141 minute

[10]

VRAAG 5

5.1 $\frac{\binom{13}{5} \times 4}{\binom{52}{5}} = 0,00198$ **of** $\frac{13}{52} \times \frac{12}{51} \times \frac{11}{50} \times \frac{9}{49} \times \frac{8}{48} \times 4$ (8)

5.2 $\frac{\binom{4}{3} \binom{4}{2} \times 13 \times 12}{\binom{52}{5}} = 0,00144$ (8)

[16]

VRAAG 6

$$6.1 \quad \int_0^3 (3kx^2 - kx^3) dx = 1$$

$$\left[kx^3 - \frac{k}{4}x^4 \right]_0^3 = 1$$

$$27k - \frac{81}{4}k = 1$$

$$108k - 81k = 4$$

$$27k = 4$$

$$k = \frac{4}{27}$$

(8)

$$6.2 \quad f(x) = \frac{4}{9}x^2 - \frac{4}{27}x^3$$

$$f'(x) = \frac{8}{9}x - \frac{4}{9}x^2$$

$$8x - 4x^2 = 0$$

$$2x - x^2 = 0$$

$$x(x - 2) = 0$$

$$x = 2$$

Modus is 2

(8)

[16]

VRAAG 7

7.1 Laat μ_1 die gemiddelde lewensduur van *Dura Power*-batterie wees.

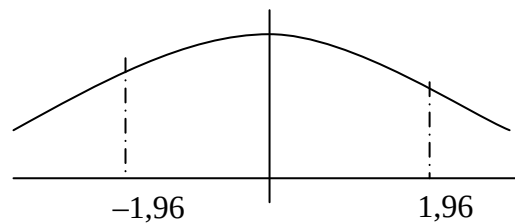
Laat μ_2 die gemiddelde lewensduur van *Long Life*-batterie wees.

$$H_0 : \mu_1 = \mu_2$$

$$H_1 : \mu_1 \neq \mu_2$$

$$\sigma_1 = 20 \quad n_1 = 100 \quad \sigma_2 = 19 \quad n_2 = 100$$

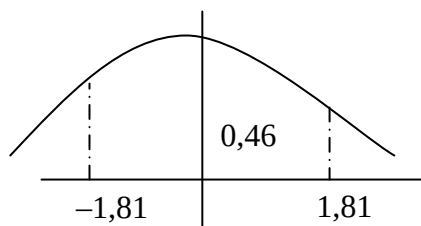
$$z = \frac{58,5 - 53,5}{\sqrt{\frac{400}{100} + \frac{361}{100}}} = 1,81$$



Daar is onvoldoende bewys by die 5%-betekenispeil om die nulhipotese te verwerp. Onvoldoende bewys van 'n verskil in die gemiddelde lewensduur van die twee soorte batteriee.

(11)

7.2



$$Z > 1,81$$

$$0,5 - 0,4649$$

$$= 0,0351$$

Tweekantige 7,02%-betekenispeil

(6)

[17]

VRAAG 8

$$\binom{10}{5} \binom{5}{4} \binom{1}{1} + \binom{10}{4} \binom{6}{4} \binom{2}{2} + \binom{10}{5} \binom{5}{3} \binom{2}{2} = 6\,930$$

[9]

Totaal vir Module 2: 100 punte

MODULE 3 FINANSIES EN MODELLERING

VRAAG 1

$$1.1 \quad Fv = \frac{3200 \left[\left(1 + \frac{0,074}{12} \right)^{30} - 1 \right]}{\frac{0,074}{12}} = \mathbf{R105\,099,30} \quad (6)$$

$$1.2 \quad 105\,000 = \frac{x \left(1 + \frac{0,074}{12} \right) \left[\left(1 + \frac{0,074}{12} \right)^{30} - 1 \right]}{\frac{0,074}{12}}$$

$$x = \mathbf{R\,3\,177,38} \quad (8)$$

[14]

VRAAG 2

$$2.1 \quad 150\,000 \left(1 + \frac{0,0625}{12} \right)^3 = \mathbf{R152\,355,98} \quad (4)$$

$$2.2 \quad 152\,355,98 - 150\,000 = \mathbf{R2\,355,98}$$

$$3\,400 - 2\,355,98 = \mathbf{R1\,044,02} \quad (4)$$

$$2.3 \quad T_n = T_{n-1} \left(1 + \frac{0,0625}{12} \right) - 3400$$

$$T_0 = 150\,000 \quad \left(1 + \frac{0,0625}{12} \right)^2 = 151\,566,57$$

$$\mathbf{T_{46} = 16\,268,76}$$

OF

$$150\,000 \left(1 + \frac{0,0625}{12} \right)^{48} - \frac{3400 \left[\left(1 + \frac{0,0625}{12} \right)^{46} - 1 \right]}{\frac{0,0625}{12}}$$

$$= 192\,478,89 - 176\,210,13 = \mathbf{16\,268,76} \quad (10)$$

$$2.4 \quad 10\,500 \left(1 + \frac{0,0625}{12} \right)^6 + 5\,000 \left(1 + \frac{0,0625}{12} \right)^4 = \mathbf{15\,937,41}$$

$$15\,937,41 < 16\,268,76 \quad \mathbf{\text{sal nie oortrokke wees nie}} \quad (10)$$

[28]

VRAAG 3

$$36\,000 = \frac{4\,000 [1 - (1 + 0,082)^{-n}]}{0,082}$$

$$1,082^{-n} = 0,262$$

$$n = 16,995 \approx 17 \text{ betalings}$$

[8]

VRAAG 4

4.1 (a) Logistiese model : 'n dravermoë bestaan (2)

(b) Fase 1: grootste verandering op y-as bestaan (2)

(c) Populasie nader dravermoë , begin gevolglik stabiliseer (2)

4.2 (a) $L_{n+1} = L_n + 0,104 \cdot L_n \left(1 - \frac{L_n}{20}\right) - 1$, $L_0 = 8$
 $L_6 = 4,75$ populasie is besig om uit te sterf. (8)

(b) $L_{n+1} = L_n + 0,104 \cdot L_n \left(1 - \frac{L_n}{20}\right) - 0,5$, $L_0 = 8$
 $L_6 = 7,995$ populasie is stabiel. (4)

4.3 $I = \text{geboortesifer} - \text{sterftesifer} = 1 \times 3 \times 0,5 \times \text{oorlewing} - 1/6 = 0,104$

Oorlewing = 18% (6)
[24]

VRAAG 5

5.1 (a) ewewig : plat af (2)

(b) 70 – 80 jaar (2)

(c) Nie 'n goeie idee om uit te dun nie:
 langtermynprojeksie is dat daar by 200 jaar baie min robbe sal wees.
 uitdunning kan tot langtermynuitsterwing lei
 OF
 Goeie idee om uit te dun:
 vir volgende eeu geen ware impak op populasie
 eeu is 'n lang tyd: baie kan gebeur om getalle weer te laat toeneem (2)

(d) Nie realisties nie, want by ewewig is daar meer haaie as robbe
 OF
 Kan realisties wees, aangesien robbe nie die uitsluitlike dieet van haaie is nie;
 nie 'n ingeslote omgewing nie (2)

- 5.2 (a) $C = \frac{1}{0,033\ 333} \therefore \text{leef tyd} = 30 \text{ jaar}$ (2)
- (b) getal robbe per geboortesiklus doorgemaak (2)
- (c) koers van dodelike interaksie vir robbe met haaie (2)
- (d) $33\ 000 = 30\ 000 + 0,38 \times 30\ 000 (1 - 30\ 000 / 44\ 000) - b \times 30\ 000 \times 30$
- b = 0,0007** (6)
- [20]**

VRAAG 6

6. 6.1 $x^2 - x - 1 = 0$ (2)
- 6.2 $x_1 = 1,61803$
 $x_2 = -0,61803$
 verhouding neig na 1,61803 (4)
- [6]**

Totaal vir Module 3: 100 punte

MODULE 4 MATRIKSE EN GRAFIEKTEORIE

VRAAG 1

1.1 (a) refleksie in die lyn $y = x$ (2)

(b) strek van faktor -3 , parallel aan x -as (y -as invariant)

OF

Refleksie in die y -as gevolg deur strek faktor 3 , parallel aan x -as (y -as invariant) (4)

1.2 (a) $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$ (2)

(b) $\begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}$ skuiwing van faktor 2 , parallel aan y -as (4)

[12]

VRAAG 2

2.1 $360^\circ \div 5 = 72^\circ$ (2)

2.2 $\begin{pmatrix} \cos 72 & -\sin 72 \\ \sin 72 & \cos 72 \end{pmatrix} \begin{pmatrix} 26,4 \\ 19,1 \end{pmatrix} = \begin{pmatrix} -10 \\ 31 \end{pmatrix}$ **E(-10; 31)** (6)

2.3 $\begin{pmatrix} \sqrt{k} & 0 \\ 0 & \sqrt{k} \end{pmatrix}$ (2)

2.4 $\begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix} \begin{pmatrix} 26,4 & 26,4 \\ 19,1 & -19,1 \end{pmatrix} = \begin{pmatrix} 31 & 19,2 \\ -10 & 26,3 \end{pmatrix}$

$$26,4 \cos 2\theta + 19,1 \sin 2\theta = 31$$

$$\text{OF } 26,4 \cos 2\theta - 19,1 \sin 2\theta = 19,2$$

$$-19,1 \cos 2\theta + 26,4 \sin 2\theta = -10$$

$$\text{OF } 19,1 \cos 2\theta + 26,4 \sin 2\theta = 26,3$$

$$\cos 2\theta = 0,950 \dots \text{ en } \sin 2\theta = 0,308 \dots$$

$$\theta = 9^\circ$$

OF

$$\begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix} = \begin{pmatrix} 31 & 19,2 \\ -10 & 26,3 \end{pmatrix} \begin{pmatrix} 26,4 & 26,4 \\ 19,1 & -19,1 \end{pmatrix}^{-1}$$

$$\begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix} = -0,000991 \begin{pmatrix} 31 & 19,2 \\ -10 & 26,3 \end{pmatrix} \begin{pmatrix} -19,1 & -26,4 \\ -19,1 & 26,4 \end{pmatrix}$$

$$\begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix} = \begin{pmatrix} 0,950 & 0,309 \\ 0,309 & -0,950 \end{pmatrix}$$

$$\therefore \cos 2\theta = 0,950$$

$$\therefore \theta = 9^\circ$$

(12)

[22]

VRAAG 3

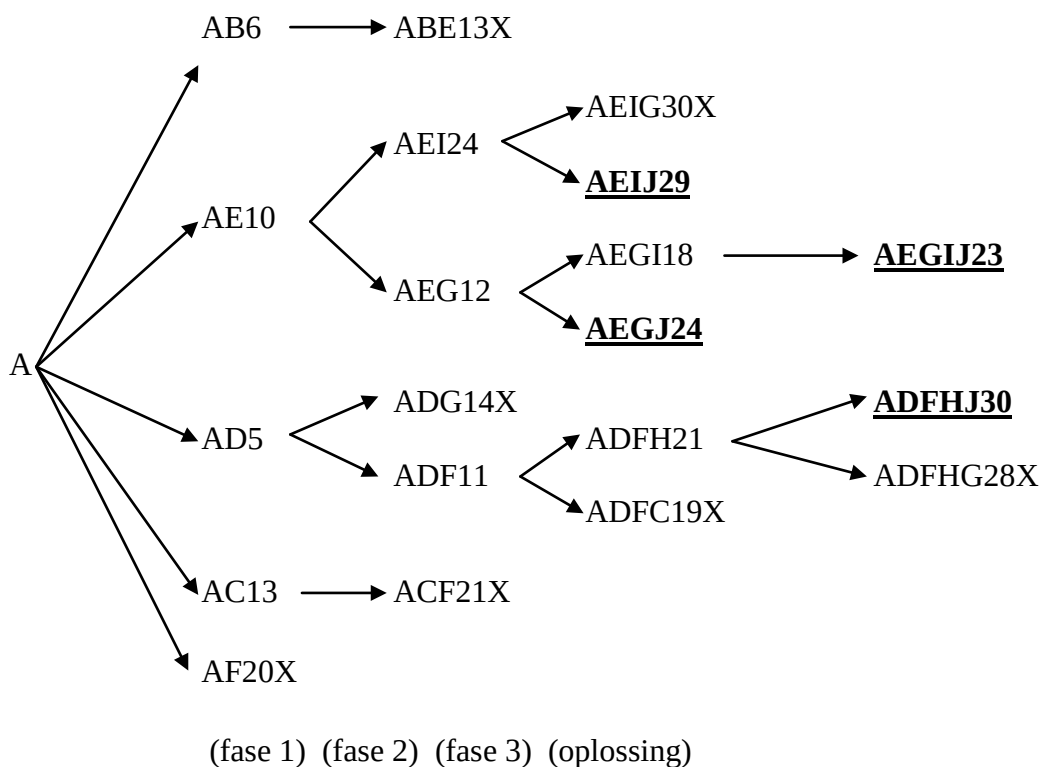
3.1 $\det P = -1(12 - 2) - 1(6 + 2) + 5(4 + 8) = 42$

$$\begin{pmatrix} 10 & 8 & 12 \\ -7 & 7 & 0 \\ -19 & -11 & -6 \end{pmatrix} \rightarrow \begin{pmatrix} 10 & -8 & 12 \\ 7 & 7 & 0 \\ -19 & 11 & -6 \end{pmatrix} \rightarrow \begin{pmatrix} 10 & 7 & -19 \\ -8 & 7 & 11 \\ 12 & 0 & -6 \end{pmatrix}$$

$$\frac{1}{42} \begin{pmatrix} 10 & 7 & -19 \\ -8 & 7 & 11 \\ 12 & 0 & -6 \end{pmatrix} \quad (10)$$

3.2 <table border="0" style="margin-left: 20px;"> <tr><td>-2</td><td>0</td><td>1</td><td>5</td></tr> <tr><td>1</td><td>-1</td><td>-3</td><td>-15</td></tr> <tr><td>1</td><td>1</td><td>-2</td><td>6</td></tr> <tr><td colspan="4"> </td></tr> <tr><td>-2</td><td>0</td><td>1</td><td>5</td></tr> <tr><td>0</td><td>-2</td><td>-5</td><td>-25</td></tr> <tr><td>0</td><td>2</td><td>-3</td><td>17</td></tr> <tr><td colspan="4"> </td></tr> <tr><td>-2</td><td>0</td><td>1</td><td>5</td></tr> <tr><td>0</td><td>-2</td><td>-5</td><td>-25</td></tr> <tr><td>0</td><td>0</td><td>-8</td><td>-8</td></tr> </table> $-8z = -8 \quad z = 1$ $-2y - 5(1) = -25 \quad y = 10$ $-2x + (1) = 5 \quad x = -2$	-2	0	1	5	1	-1	-3	-15	1	1	-2	6					-2	0	1	5	0	-2	-5	-25	0	2	-3	17					-2	0	1	5	0	-2	-5	-25	0	0	-8	-8	OF <table border="0" style="margin-left: 20px;"> <tr><td>1</td><td>-1</td><td>-3</td><td>-15</td></tr> <tr><td>1</td><td>1</td><td>-2</td><td>6</td></tr> <tr><td>-2</td><td>0</td><td>1</td><td>5</td></tr> <tr><td colspan="4"> </td></tr> <tr><td>1</td><td>-1</td><td>-3</td><td>-15</td></tr> <tr><td>0</td><td>-2</td><td>-1</td><td>-21</td></tr> <tr><td>0</td><td>-2</td><td>-5</td><td>-25</td></tr> <tr><td colspan="4"> </td></tr> <tr><td>1</td><td>-1</td><td>-3</td><td>-15</td></tr> <tr><td>0</td><td>-2</td><td>-1</td><td>-21</td></tr> <tr><td>0</td><td>0</td><td>4</td><td>4</td></tr> </table> $4z = 4 \quad z = 1$ $-2y - (1) = -21 \quad y = 10$ $1x - (10) - 3(1) - 15 \quad x = -2$	1	-1	-3	-15	1	1	-2	6	-2	0	1	5					1	-1	-3	-15	0	-2	-1	-21	0	-2	-5	-25					1	-1	-3	-15	0	-2	-1	-21	0	0	4	4
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(10)
[20]

VRAAG 4

[10]

VRAAG 5

5.1	CG	65	CG	65	CG	65	
	JD	90	CP	100	JG	100	
	JG	100	GJ	100	JD	90	
	CP	100	JD	90	CP	100	
	JW	120	WJ	120	WJ	120	
	+	155	+	155	+	155	
		630		630		630	(8)

5.2	CG	65										
	CB	95										
	JB	70										
	JD	90										
	DJP	90 + 110										
	PJW	110 + 120										
	WCG	135 + 65										
		950 km										(8)

5.3	Nee , lengte van tweede helfte is aansienlik groter as dié van eerste helfte.											(2)
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5.4	900	G	J	P	C	B	D	J	W	C	G	
	880	G	C	W	J	B	D	C	P	C	G	
	Enige roete wat alle nodusse gebruik, waar $630 < x < 950$											(8)
	afstand roete wat al sewe nodusse gebruik											
	– vir elke nodus uitgelaat											

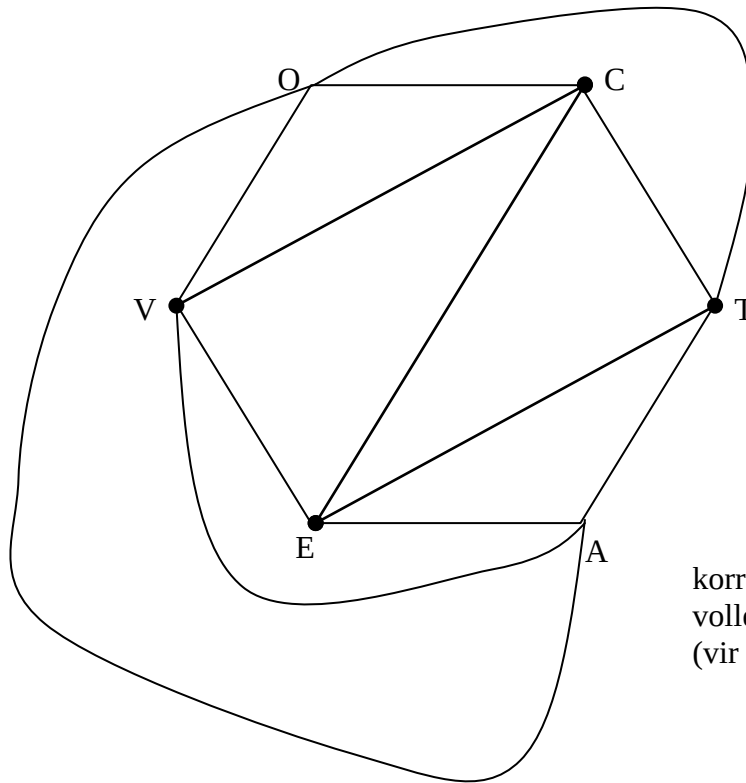
[26]

VRAAG 6

6.1 Ja , alle hoekpunte het dieselfde graad. (2)

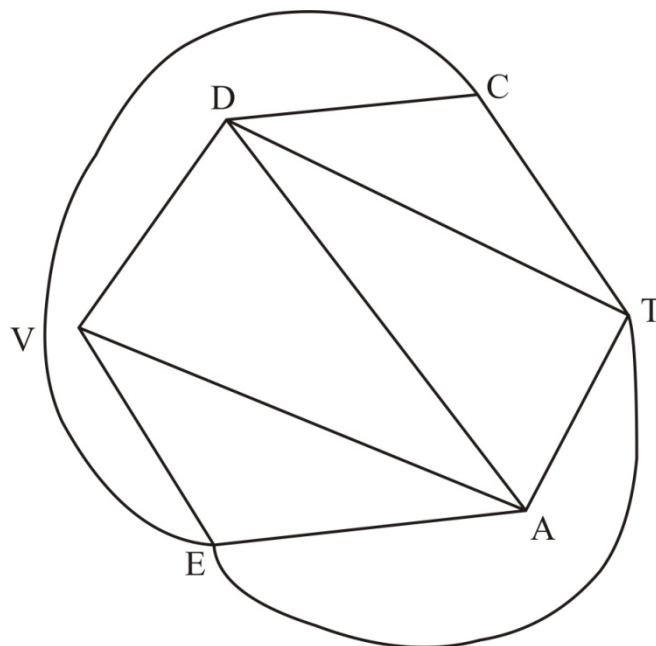
6.2 Ja , alle hoekpunte het ewe grade. (2)

6.3



korrek en
volledig
(vir elke korrekte kant geskuif)

OF



(6)
[10]

Totaal vir Module 4: 100 punte

Totaal: 300 punte