



GRAAD 12-EKSAMEN
NOVEMBER 2012

GEVORDERDEPROGRAM-WISKUNDE

NASIENRIGLYNE

Tyd: 3 uur

300 punte

Hierdie nasienriglyne is voorberei vir gebruik deur eksaminatore en subeksaminatore, wat 'n standaardisasievergadering moet bywoon ten einde te verseker dat die riglyne konsekwent geïnterpreteer word en gedurende die nasien van kandidate se skrifte toegepas word.

Die IEB sal nie enige gesprek of korrespondensie voer oor enige nasienriglyne nie. Daar word besef dat daar verskillende sienings oor sekere sake van beklemtoning of besonderhede in die riglyne kan wees. Daar word ook besef dat sonder die voordeel van die bywoning van 'n standaardisasievergadering, daar verskillende interpretasies van die toepassing van die nasienriglyne kan wees.

MODULE 1 CALCULUS EN ALGEBRA**VRAAG 1**

RTP: $9^n - 8n - 1 = 8p \quad \forall n \in \mathbb{N}; n \geq 2$

Stap 1: Vir $n = 2$ waar omdat $9^2 - 16 - 1 = 64 = 8^2$

Stap 2: Veronderstel waar vir $n = k$, d.w.s. $9^k - 8k - 1 = 8q$ waar $q \in \mathbb{N}$

Stap 3: vir $n = k + 1$

$$\begin{aligned} \text{LHS} &= 9^{(k+1)} - 8(k+1) - 1 \\ &= 9 \cdot 9^k - 8k - 8 - 1 \\ &= 9^k - 8k - 1 + 8 \cdot 9^k - 8 \\ &= 8q + 8(9^k - 1) \\ &= 8(q + 9^k - 1) \end{aligned}$$

$\therefore$ Bewering waar $\forall n \in \mathbb{N}; n \geq 2$ volgens die beginsel van MI

[14]**VRAAG 2**

$$2.1 \quad \log x = \log 2 + \log(x+5) - \log(x-1)$$

$$\therefore \log x = \log \left(\frac{2(x+5)}{x-1} \right)$$

$$\therefore x(x-1) = 2x+10$$

$$\therefore x^2 - 3x - 10 = 0$$

$$\therefore (x-5)(x+2) = 0$$

$$x = 5 \text{ maar } x \neq -2$$

(6)

$$2.2 \quad (a) \quad e^{(x)} \text{ is stygend } \therefore f(x) = \frac{1}{1+e^x} \text{ is dalend}$$

(3)

$$(b) \quad \text{Die waardeversameling van } f(x) \text{ is } (0; 1)$$

(4)

$$(c) \quad \text{Die inverse funksie:}$$

$$x = \frac{1}{1+e^y}$$

$$\therefore 1+e^y = \frac{1}{x}$$

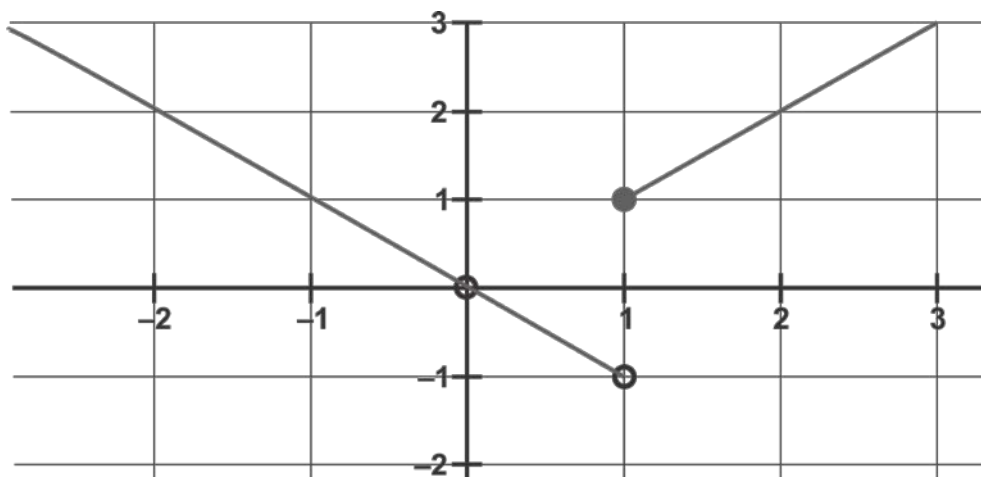
$$\therefore e^y = \frac{1}{x} - 1$$

$$\therefore y = \ln\left(\frac{1}{x} - 1\right)$$

$$\therefore f^{-1}(x) = \ln\left(\frac{1}{x} - 1\right) \text{ of } \ln\left(\frac{1-x}{x}\right)$$

(4)

2.3 (a)



(4)

(b) Definisieversameling van $g(f(x))$ is $(-\infty; 0) \cup [1; \infty)$

(4)

[25]

VRAAG 3

3.1 $(2+i)(a+bi+3i)=8i+6$

OF $(2a-b-3)+(2b+6+a)i=6+8i$

$$\begin{aligned} \therefore a+i(b+3) &= \frac{8i+6}{2+i} \times \frac{2-i}{2-i} \\ &= \frac{16i-8i^2+12-6i}{4-i^2} \\ &= \frac{20+10i}{5} \end{aligned}$$

$$\begin{aligned} \therefore 2a-b-3 &= 6 \text{ en } 2b+6+a=8 \\ \therefore 2a-b &= 9 \\ a+2b &= 2 \end{aligned} \left. \vphantom{\begin{aligned} \therefore 2a-b &= 9 \\ a+2b &= 2 \end{aligned}} \right\} \text{los op gelyktydig}$$

$$\begin{aligned} \therefore -5b &= 5 \text{ en } a=4 \\ b &= -1 \end{aligned}$$

$$a+i(b+3)=4+2i$$

$$\therefore a=4 \text{ en } b=-1$$

(7)

3.2 $x^3-5x^2+kx-13=0$ en $x=2+3i$

$$(x-2)^2-(3i)^2=x^2-4x+4+9=x^2-4x+13$$

$$\text{Dus } (x^2-4x+13)(x-1)=0 \quad x^3-5x^2+17x-13=0$$

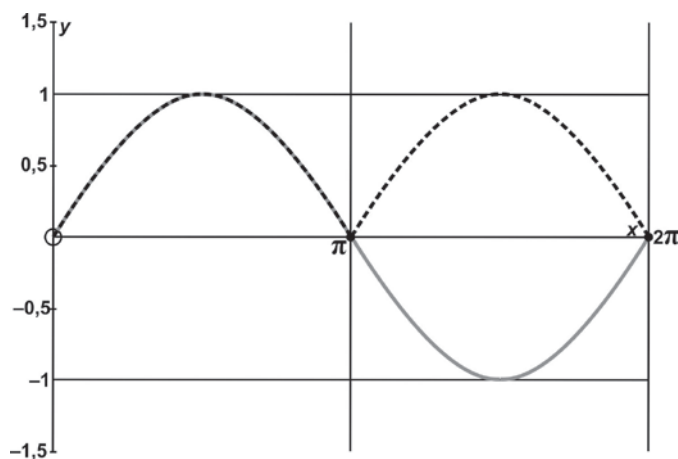
$$\text{Dus } k=17$$

(8)

[15]

VRAAG 4

4.1 (a)



(8)

- (b) $|\sin x| \geq 0 \forall x \in \mathbb{R}$ maar $-1 \leq \sin|x| \leq 1$ aangesien daar waardes van $x \geq 0$ bestaan waar $\sin x < 0$

(2)

$$4.2 \quad (a) \quad (i) \quad f(x) = \begin{cases} px & \text{as } x < \pi \\ -2\cos x & \text{as } \pi \leq x \leq \frac{3\pi}{2} \\ \frac{1}{\pi} \left(x - \frac{5\pi}{2} \right)^2 - 2 & \text{as } x > \frac{3\pi}{2} \end{cases}$$

Vir f om kontinu te wees by $x = \pi$ moet $p\pi = -2\cos\pi$

Dus $p\pi = 2$

$$\therefore p = \frac{2}{\pi}$$

(4)

$$(ii) \quad f\left(\frac{3\pi}{2}\right) = -2\cos\frac{3\pi}{2} = 0$$

Maar

$$\lim_{x \downarrow \frac{3\pi}{2}} f(x) = \frac{1}{\pi} \left(\frac{3\pi}{2} - \frac{5\pi}{2} \right)^2 - 2 = \pi - 2$$

$$\therefore f \text{ nie kontinu by } x = \frac{3\pi}{2}$$

(5)

(b) (i) die gradiënt van f as $x \rightarrow \pi^-$ is $\frac{2}{\pi}$ wat konstant is (reguitlyn)

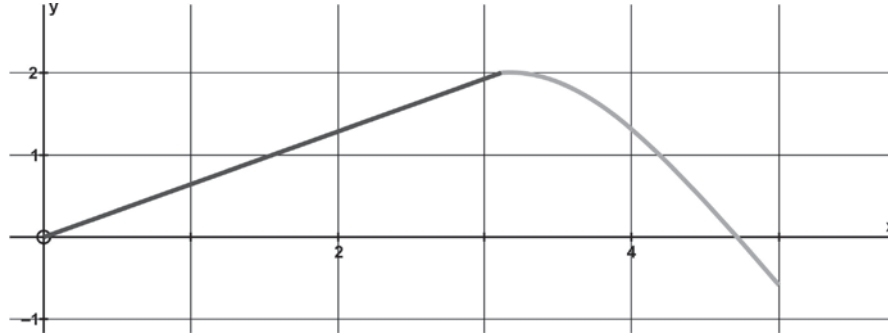
die gradiënt van f as $x \rightarrow \pi^+$ is $-2(-\sin\pi) = 0$

∴ die gradiënt van links $\neq$ gradiënt van regs

∴ alhoewel kontinu

nie differensieerbaar

Alternatief:



Sien duidelik die gradiënt van regs $\neq$ gradiënt van links by π (5)

$$(ii) \quad \lim_{x \rightarrow \left(\frac{3\pi}{2}\right)^-} f'(x) = -2 \left(-\sin \frac{3\pi}{2} \right) = -2$$

$$\lim_{x \rightarrow \left(\frac{3\pi}{2}\right)^+} f'(x) = \frac{2}{\pi} \left(\frac{3\pi}{2} - \frac{5\pi}{2} \right) = -2 \quad (6)$$

(iii) Nee, want alhoewel hulle dieselfde gradiënt het, is die funksie diskontinu by $x = \frac{3\pi}{2}$.

(2)
[32]

VRAAG 5

5.1 (a) Jy kan nie die log van 'n negatiewe getal kry nie. (2)

(b) $\ln x = 0$

$\therefore x = e^0 = 1$ (2)

(c)
$$\frac{d}{dx} \left(\frac{x}{\ln x} \right) = \frac{1 \cdot \ln x - \frac{1}{x} \cdot x}{(\ln x)^2} = \frac{\ln x - 1}{(\ln x)^2}$$

Dus draaipunt wanneer $\ln x - 1 = 0$ wat is wanneer $x = e$ wat $y = \frac{e}{\ln e} = e$ gee.

Dus koördinate van DP is $(e; e)$ (8)

5.2 (a) Skuins asimptoot: $2x^2 - 2x + 5 = (x+1)(2x-4) + 9$

Dus: $\frac{2x^2 - 2x + 5}{x+1} = 2x - 4 + \frac{9}{x+1}$ dus die skuins asimptoot is $y = 2x - 4$ (6)

(b) Nee. Om te raak $2x - 4 + \frac{9}{x+1} = 2x - 4$

$\therefore \frac{9}{x+1} = 0$ wat nie moontlik is nie. (3)

[21]

VRAAG 6

Breedte $= \frac{3}{n}$ en $x_i = \frac{3i}{n}$

Benaderde oppervlakte $= \sum_{i=1}^n \left(3 \left(\frac{3i}{n} \right)^2 + 1 \right) \cdot \frac{3}{n}$

$= \sum_{i=1}^n \left(\frac{81i^2}{n^3} + \frac{3}{n} \right)$

$= \frac{81}{n^3} \sum_{i=1}^n i^2 + \frac{3}{n} \sum_{i=1}^n 1$

$= \frac{81}{n^3} \left(\frac{n^3}{3} + \frac{n^2}{2} + \frac{n}{6} \right) + \frac{3}{n} \cdot n$

$= 27 + \frac{81}{2n} + \frac{81}{6n^2} + 3$

$\therefore \text{oppervlakte} = \lim_{n \rightarrow \infty} \left(27 + \frac{81}{2n} + \frac{81}{6n^2} + 3 \right) = 30$

[14]

VRAAG 7

$$\begin{aligned}
 7.1 \quad r^2 &= (r-6)^2 + 15^2 \\
 \therefore 12r &= 261 \\
 \therefore r &= 21,75 \text{ cm}
 \end{aligned} \tag{6}$$

$$\begin{aligned}
 7.2 \quad \sin \frac{\theta}{2} &= \frac{15}{21,75} \\
 \therefore \frac{\theta}{2} &= 0,761 \\
 \therefore \theta &= 1,522 \\
 \text{Dus is die booglengte ACB} &= 21,75 \times 1,522 = 33,1041 \text{ cm}
 \end{aligned} \tag{5}$$

$$\begin{aligned}
 7.3 \quad \text{Oppervlakte van segment} \\
 = 0,5 \times (21,75)^2 \times 1,522 - 0,5 \times (21,75)^2 \sin 1,522 &= 123,7566 \text{ cm}^2
 \end{aligned} \tag{5}$$

[16]

VRAAG 8

$$8.1 \quad \text{LHS: } 2^3 - 3 \cdot 2 \cdot (-1)^2 + (-1)^3 = 8 - 6 - 1 = 1 = \text{RHS} \tag{2}$$

$$\begin{aligned}
 8.2 \quad 3x^2 - 3 \left(y^2 + x \cdot 2y \cdot \frac{dy}{dx} \right) + 3y^2 \frac{dy}{dx} &= 0 \\
 \therefore \frac{dy}{dx} (-6xy + 3y^2) &= -3x^2 + 3y^2 \\
 \therefore \frac{dy}{dx} &= \frac{x^2 - y^2}{2xy - y^2}
 \end{aligned} \tag{8}$$

$$\begin{aligned}
 8.3 \quad \text{by } (2; -1) \frac{dy}{dx} &= \frac{4 - (-1)^2}{2(2)(-1) - (-1)^2} = \frac{-3}{5} \\
 \therefore y &= -\frac{3}{5}x + c \\
 -1 &= -\frac{3}{5} \cdot 2 + c \\
 \therefore c &= \frac{1}{5} \\
 \therefore y &= -\frac{3}{5}x + \frac{1}{5}
 \end{aligned} \tag{5}$$

[15]

VRAAG 9

$$9.1 \quad \int 2x^3 - \sec^2\left(\frac{x}{2}\right) - \frac{1}{\sqrt[3]{x}} dx = \frac{1}{2}x^4 - 2\tan^2\left(\frac{x}{2}\right) - \frac{3}{2}x^{\frac{2}{3}} + c \quad (6)$$

$$9.2 \quad \int (\cos^2 3x)(\sin 3x) dx$$

Laat $\cos 3x = u$

$$\text{Dan: } \frac{du}{dx} = -\sin 3x \cdot 3 \text{ of } -\frac{du}{3} = \sin 3x dx$$

$$\text{Dus: } -\frac{1}{3} \int u^2 du = -\frac{1}{9} u^3 + c = -\frac{1}{9} \cos^3 3x + c \quad (7)$$

$$9.3 \quad \int \cos 4\theta \cdot \sin 5\theta d\theta = \frac{1}{2} \int \sin 9\theta + \sin(\theta) d\theta$$

$$= -\frac{1}{2} \cdot \frac{\cos 9\theta}{9} - \frac{1}{2} \cdot \cos \theta + c \quad (6)$$

$$9.4 \quad \int y \sqrt{y+3} dy$$

$$\text{Laat } u = y+3 \text{ dan } \frac{du}{dy} = 1 \text{ of } du = dy$$

$$\text{Dus } \int y \sqrt{y+3} dy = \int (u-3) \cdot u^{\frac{1}{2}} du = \int u^{\frac{3}{2}} - 3u^{\frac{1}{2}} du$$

$$= \frac{2}{5} u^{\frac{5}{2}} - 2u^{\frac{3}{2}} + c = \frac{2}{5} (y+3)^{\frac{5}{2}} - 2(y+3)^{\frac{3}{2}} + c$$

OF

$$\text{laat } f(xg) = y \quad \therefore f'(y) = 1$$

$$g'(eg) = \sqrt{y+3} \quad \therefore g(y) = \frac{2}{3} (y+3)^{\frac{3}{2}}$$

$$\therefore \int fg' dy = fg - \int f' g dy$$

$$= \frac{2y}{2} (y+3)^{\frac{3}{2}} - \frac{2}{3} \int (y+3)^{\frac{3}{2}} dy$$

$$= \frac{2y}{3} (y+3)^{\frac{3}{2}} - \frac{22}{35} (y+3)^{\frac{5}{2}} + c$$

$$= \frac{2y}{3} (y+3)^{\frac{3}{2}} - \frac{4}{15} (y+3)^{\frac{5}{2}} + c \quad (9)$$

[28]

VRAAG 10

$$10.1 \quad Vol = \pi \int_1^a \left(\frac{1}{x}\right)^2 dx = \pi \left[-\frac{1}{x} \right]_1^a = -\frac{\pi}{a} + \pi \text{ eenhede}^3 \quad (8)$$

$$10.2 \quad Vol(a \rightarrow \infty) = \pi \text{ eenhede}^3 \quad (2)$$

[10]

VRAAG 11

$$S = \frac{8k}{x^2} + \frac{k}{(15-x)^2}$$

$$\text{Dus } \frac{dS}{dx} = -2.8kx^{-3} - 2.k(15-x)^{-3} \cdot (-1) = 0$$

$$\therefore 8(15-x)^3 - x^3 = 0$$

$$\therefore (2(15-x) - x)(4(15-x)^2 + 2x(15-x) + x^2) = 0 \quad \text{of } \therefore 8(15-x)^3 = x^3$$

$$\therefore 30 - 3x = 0 \text{ or } x = 10 \text{ km van A} \quad \therefore 2(15-x) = x$$

$$\text{Kontroleer vir min: } \frac{d^2S}{dx^2} = 48kx^{-4} + 6k(15-x)^{-4} > 0 \text{ vir } x = 10 \text{ dus minimum.} \quad \therefore x = 10$$

[10]**Totaal vir Module 1: 200 punte**

MODULE 2 STATISTIEK

VRAAG 1

1.1 $X \sim \text{bin}(20; 7)$

$$P(x=7) = \binom{20}{7} (0,2)^7 (0,8)^{13} = 0,0545 \quad (6)$$

1.2 (a) $\binom{12}{4} = 495 \quad (1)$

(b) $\binom{6}{2} \binom{6}{2} = 225 \quad (3)$

(c) $\binom{6}{1} \binom{5}{1} \times 2 = 60 \quad (4)$

1.3 $\frac{\binom{9}{3} \binom{6}{3} \binom{3}{3}}{3!} = 280 \quad (5)$

1.4 $P(\text{minstens een}) = 1 - P(\text{geen})$

$$= 1 - \frac{\binom{4}{0} \binom{16}{10}}{\binom{20}{10}} = 0,9567 \quad (7)$$

[26]

VRAAG 2

$$X \sim N(100; 15^2)$$

2.1 $P(120 < x < 129) = P\left(\frac{120-100}{15} < z < \frac{129-100}{15}\right)$
 $= P(1,33 < z < 1,93)$
 $= 0,4732 - 0,4082$
 $= 0,065$
 $\therefore 6,5\% \quad (10)$

2.2 $P(X < a) = 0,022$

$$-2,01 = \frac{a-100}{15}$$

$$a = 69,5$$

$$a \approx 70$$

$$\therefore \text{minder as } 70\% \quad (6)$$

[16]

VRAAG 3

$$\begin{aligned} 3.1 \quad \bar{x} &= 57,8\% \\ \bar{y} &= 60,7\% \end{aligned} \quad (2)$$

$$\begin{aligned} 3.2 \quad b &= \frac{n(\sum xy) - (\sum x)(\sum y)}{n\sum x^2 - (\sum x)^2} = \frac{10(39070) - (578)(607)}{10(38378) - (578)^2} \\ &= 0,802 \\ &= \text{sub}(57,8 ; 60,7) \\ 60,7 &= a + 0,802(57,8) \\ a &= 14,344 \\ \therefore y &= 14,344 + 0,802x \end{aligned} \quad (6)$$

$$3.3 \quad r = \frac{10(39070) - (578)(607)}{\sqrt{[10(38378) - (578)^2][10(41221) - (607)^2]}} = 0,8546 \quad (3)$$

$$\begin{aligned} 3.4 \quad \text{Allie, } y &= 14,344 + 0,802(85) \\ &= 82,5 \end{aligned} \quad (3)$$

$$3.5 \quad \text{Dit is 'n betroubare beraming aangesien daar 'n sterk korrelasie bestaan.} \quad (2)$$

[16]

VRAAG 4

$$4.1 \quad (a) \quad 2010: \bar{x} = 62 \quad \sigma = 49,452 \quad 2011: \bar{y} = 82,4 \quad \sigma = 96,8055 \quad (4)$$

$$(b) \quad H_0: \mu_y - \mu_x \geq 10$$

$$H_1: \mu_y - \mu_x < 10$$

Verwerpingsgebied:

Verwerp H_0 if $Z > 1,28$

$$\text{Toetsstatistiek: } z = \frac{(82,4 - 62) - (10)}{\sqrt{\frac{(96,8055)^2}{5} + \frac{(49,452)^2}{5}}} = 0,2139$$

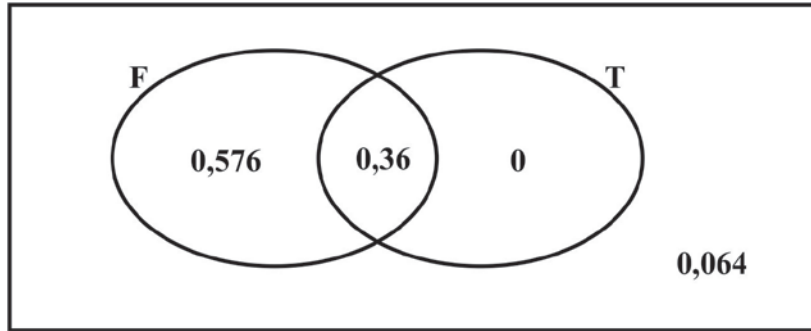
Gevolgtrekking: Ons slaag nie daarin om H_0 by die 10%-betekenispeil te verwerp nie en stel onvoldoende bewys voor om die bewering te staaf. (10)

$$\begin{aligned} 4.2 \quad 1,88\sqrt{\frac{(0,25)(0,75)}{n}} &< 0,04 \\ \sqrt{\frac{(0,25)(0,75)}{n}} &< \frac{1}{47} \\ \frac{3}{16n} &< \frac{1}{2209} \\ 414,1 &< n \quad n = 415 \end{aligned} \quad (6)$$

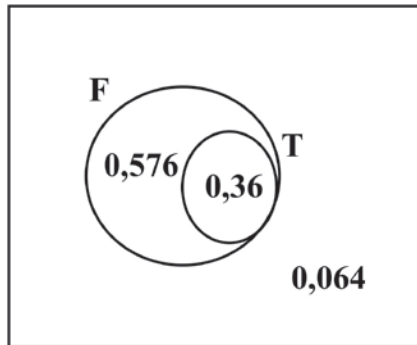
[20]

VRAAG 5

$$\begin{aligned}
 5.1 \quad P(F|T) &= 1 & P(F|T') &= 0,9 & P(T) &= 0,36 \\
 P(F|T) &= \frac{P(F \cap T)}{P(T)} & P(F|T') &= \frac{P(F \cap T')}{P(T')} \\
 0,36 &= P(F \cap T) & (0,9)(0,64) &= P(F \cap T') \\
 & & 0,576 &= P(F \cap T')
 \end{aligned}$$



OF



(10)

$$\begin{aligned}
 5.2 \quad (a) \quad P(F) &= 0,576 + 0,36 = 0,936 & (2) \\
 (b) \quad P(F' \cap T') &= 1 - 0,936 = 0,064 & (1) \\
 & & [13]
 \end{aligned}$$

VRAAG 6

$$\begin{aligned}
 6.1 \quad \text{Alicia: } z &= \frac{98 - 56}{18} = 2,33 \\
 \text{Rae: } z &= \frac{40 - 56}{18} = -0,89 & (2) \\
 6.2 \quad \text{Alicia: } 2,33 &= \frac{x - 60}{12} \\
 x &= 87,99 \\
 \text{Rae: } -0,89 &= \frac{x - 60}{12} \\
 x &= 49,32 & (6) \\
 6.3 \quad \text{Standaardafwyking is verlaag om die wye omvang van waardes te verminder, om} & \\
 \text{meer eenvormigheid te bewerkstellig.} & (1)
 \end{aligned}$$

[9]

Totaal vir Module 2: 100 punte

MODULE 3 FINANSIES EN MODELLERING**VRAAG 1**

$$1.1 \quad 5\,600 \left(1 + \frac{0,0684}{4}\right)^{20} = 7\,860,70$$

$$7\,860,70 - 5\,600 = \mathbf{2\,260,70} \quad (4)$$

$$1.2 \quad 10\,033,38 = x \frac{\left[\left(1 + \frac{0,0684}{4}\right)^{20} - 1\right]}{\frac{0,0684}{4}} \quad x = \mathbf{425} \quad (6)$$

$$1.3 \quad \left(1 + \frac{0,0684}{12}\right)^{12} = \left(1 + \frac{i}{4}\right)^4$$

$$FV = \frac{420 \left[\left(1 + \frac{0,0688}{4}\right)^{20} - 1\right]}{\frac{0,0688}{4}} = \mathbf{9\,925,16} \quad (10)$$

Gebruik $i = 0,069$: $x = 9\,930,07$

Gebruik $i = 0,06879$: $x = 9\,924,91$

Gebruik $i = 0,07$: $x = 9\,954,68$

[20]**VRAAG 2**

$$(1 + 0,12)^2 (1 + 0,105) (1 + i)^3 = (1 + 0,1)^6$$

$$i = \mathbf{0,0852236 = 8,52\%}$$

[8]

VRAAG 3

$$\begin{aligned}
 3.1 \quad OB &= 850\,000 \left(1 + \frac{0,0856}{12}\right)^{80} - \frac{7\,400 \left[\left(1 + \frac{0,0856}{12}\right)^{80} - 1 \right]}{\frac{0,0856}{12}} \\
 &= 1\,500\,985,64 - 794\,495,95 = \mathbf{706\,489,70} \quad (8)
 \end{aligned}$$

$$\begin{aligned}
 3.2 \quad 12\,000 \left(1 + \frac{0,0856}{12}\right)^{63} + 12\,000 \left(1 + \frac{0,0856}{12}\right)^{15} &= 32\,128,62 \\
 706\,489,70 - 32\,128,62 &= \mathbf{674\,361,08} \quad (8)
 \end{aligned}$$

$$\begin{aligned}
 3.3 \quad 675\,000 \left(1 + \frac{0,0856}{12}\right)^{159} - \frac{7\,100 \left[\left(1 + \frac{0,0856}{12}\right)^{159} - 1 \right]}{\frac{0,0856}{12}} \\
 = 2\,089\,931,38 - 2\,086\,399,34 = 3\,532,04 \\
 3\,532,04 \left(1 + \frac{0,0856}{12}\right) = \mathbf{3\,557,24}
 \end{aligned}$$

OF

$$\begin{aligned}
 675\,000 \left(1 + \frac{0,0856}{12}\right)^{160} - \frac{7\,100 \left(1 + \frac{0,0856}{12}\right) \left[\left(1 + \frac{0,0856}{12}\right)^{159} - 1 \right]}{\frac{0,0856}{12}} \\
 = 2\,104\,839,56 - 2\,101\,282,32 = \mathbf{3\,557,24}
 \end{aligned}$$

OF

$$\begin{aligned}
 675\,000 - \frac{7\,100 \left[1 - \left(1 + \frac{0,0856}{12}\right)^{-159} \right]}{\frac{0,0856}{12}} \\
 = 675\,000 - 673\,859,23 = 1\,140,77 \\
 1\,140,77 \left(1 + \frac{0,0856}{12}\right)^{160} = \mathbf{3\,557,24} \quad (10)
 \end{aligned}$$

[26]

VRAAG 4

$$4.1 \quad 3 \times 4 \times 0,6 \times 0,8 - 1/7 = 5,617 \quad (6)$$

$$4.2 \quad H_{n+1} = 6,6H_n \quad (2)$$

$$4.3 \quad H_0 = 25 \quad H_1 = 165 \quad H_2 = 1\,089 \quad (2)$$

$$4.4 \quad (1 + 5,6) = \left(1 + \frac{r}{52}\right)^{52} \quad r = 1,922 \text{ (weeklikse tempo)}$$

$$H_1 = 25,925 \quad \therefore 0,925 \approx 1 \text{ vark per week} \quad (6)$$

[16]

VRAAG 5

$$5.1 \quad (a) \quad \text{wildsbokke: } 1\,200 - 1\,400 \quad \text{luiperds: } 46 - 48 \quad (2)$$

$$(b) \quad 42 < \text{luiperds} < 50 \quad (2)$$

$$(c) \quad \text{wildsbokke} = 800 \quad (2)$$

$$(d) \quad 420 = b \times 4\,500 \times 30 \quad b = 0,00311 \quad (5)$$

$$(e) \quad f \times 0,003 \times 30 \times 4\,500 = 4 \quad f = 0,00988 \quad (5)$$

$$5.2 \quad (a) \quad \text{Beide populasies neem aanvanklik toe; voorheen het slegs luiperds toegeneem.} \quad (2)$$

$$(b) \quad \text{Variasiewydte van beide populasies is groter in tweede diagram.} \quad (2)$$

$$(c) \quad \text{Ewewigspunt sal oor langer tydperk bereik word in tweede diagram.} \quad (2)$$

[22]

VRAAG 6

$$6.1 \quad T_{n+1} = 1,333 T_n, T_1 = 27 \quad (4)$$

$$6.2 \quad T_{14} = 1\,136,495$$

OF

$$27 \left(\frac{4}{3}\right)^{n-1} > 1\,000$$

$$n = 14$$

$$27 \left(\frac{4}{3}\right)^{13} = 1\,136,495 \quad (4)$$

[8]**Totaal vir Module 3: 100 punte**

MODULE 4 MATRIKSE EN GRAFIEKTEORIE

VRAAG 1

$$1.1 \quad 2M - 3N = \begin{pmatrix} 12 & -4 \\ -6 & 2 \end{pmatrix} \checkmark - \begin{pmatrix} 15 & 6 \\ -6 & 9 \end{pmatrix} = \begin{pmatrix} -3 & -10 \\ 0 & -7 \end{pmatrix} \quad (4)$$

1.2 Nie 'n vierkantige matriks nie. (2)

1.3 Een ry is 'n veelvoud van 'n ander OF determinant is nul. (2)

$$1.4 \quad \frac{1}{19} \begin{pmatrix} 3 & -2 \\ 2 & 5 \end{pmatrix} \quad (4)$$

[12]

VRAAG 2

$$2.1 \quad \begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 2 & 2 \\ -4 & 5 \end{pmatrix} = \begin{pmatrix} -10 & 17 \\ -4 & 5 \end{pmatrix} \quad (6)$$

$$2.2 \quad \tan \theta = 3 \quad \theta = 71,565^\circ$$

$$\begin{pmatrix} \cos 2(71,565) & \sin 2(71,565) \\ \sin 2(71,565) & -\cos 2(71,565) \end{pmatrix} = \begin{pmatrix} -0,8 & 0,6 \\ 0,6 & 0,8 \end{pmatrix} \quad (6)$$

$$2.3 \quad \begin{pmatrix} 1 & 0 \\ 0 & 3 \end{pmatrix} \text{ OF } \begin{pmatrix} 3 & 0 \\ 0 & 1 \end{pmatrix} \text{ EN } \begin{pmatrix} \sqrt{3} & 0 \\ 0 & \sqrt{3} \end{pmatrix} \quad (4)$$

$$2.4 \quad \begin{pmatrix} \cos \theta & \sin \theta \\ \sin \theta & -\cos \theta \end{pmatrix} \begin{pmatrix} 25 \\ -75 \end{pmatrix} = \begin{pmatrix} -45 \\ 65 \end{pmatrix}$$

$$25\cos \theta - 75\sin \theta = -45 \quad \text{OF} \quad 25\sin \theta + 75\cos \theta = 65$$

$$\theta = 53,13^\circ \quad (8)$$

[24]

VRAAG 3

$$3.1 \quad AB = \begin{pmatrix} 1 & 2 & -3 \\ -3 & 2 & 0 \\ 2 & -1 & 4 \end{pmatrix} \begin{pmatrix} 8 & -5 & 6 \\ -12 & -10 & -9 \\ -1 & 5 & 8 \end{pmatrix} = \begin{pmatrix} -13 & -40 & -36 \\ -48 & -5 & -36 \\ 24 & 20 & 53 \end{pmatrix}$$

$$BA = \begin{pmatrix} 8 & -5 & 6 \\ -12 & -10 & -9 \\ -1 & 5 & 8 \end{pmatrix} \begin{pmatrix} 1 & 2 & -3 \\ -3 & 2 & 0 \\ 2 & -1 & 4 \end{pmatrix} = \begin{pmatrix} 35 & 0 & 0 \\ 0 & -35 & 0 \\ 0 & 0 & 35 \end{pmatrix} \quad (8)$$

$$3.2 \quad \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} \begin{pmatrix} a & d & g \\ b & e & h \\ c & f & i \end{pmatrix} = \begin{pmatrix} a^2 + b^2 + c^2 & ad + be + cf & ag + bh + ci \\ ad + be + cf & d^2 + e^2 + f^2 & dg + eh + fi \\ ag + bh + ci & dg + eh + fi & g^2 + h^2 + i^2 \end{pmatrix} \quad (10)$$

[18]

VRAAG 4

4.1 Elke nodus word een keer en slegs een keer gebruik voor terugkeer na begin. (2)

4.2

A	B	J	B	C	D	E	C	E	F	C
22	10	10	21	16	6	21	21	18	26	16
G	B	H	G	F	G	H	J	A		
22	17	7	13	13	7	13	20			

Verdubbelde skakels $BJ + FG + GH + CE = 51$

Kring met al nege nodusse en $15 + 4$ skakels

Totale gewig = 248 (oorspronklike grafiek) + 51 (verdubbelde skakels)

= **299 m** (12)

4.3 $300 = 248$ (bestaande skakels) + 35 ($BH + EF$) + CJ

$CJ = 17 m$ (10)
[24]

VRAAG 5

5.1 9 skakels (2)

5.2 Kruskal: kortste skakels in grafiek word gekies (2)

5.3 6 (2)

5.4 JE: sal 'n minikring voltooi (2)

5.5 DE, AB, IJ, DI **EF / JG** **HG / CI** **BI = 60** (8)
[16]

VRAAG 6

6.1

1	3	6	10
2	3	4	
1	1		

$\therefore n = 15$ (3)

6.2 Euler-kring $\therefore$ benodig ewe skakels by nodusse

$\therefore n \approx$ onewe getal nodusse.

(3)
[6]

Totaal vir Module 4: 100 punte

Totaal: 300 punte