



GRAAD 12-EKSAMEN
NOVEMBER 2013

GEVORDERDEPROGRAM-WISKUNDE

NASIENRIGLYNE

Tyd: 3 uur

300 punte

Hierdie nasienriglyne word voorberei vir gebruik deur eksaminatore en hulpeksaminatore. Daar word van alle nasieners vereis om 'n standaardiseringsvergadering by te woon om te verseker dat die nasienriglyne konsekwent vertolk en toegepas word tydens die bepunting van kandidate se skrifte.

Die IEB sal geen gesprek aanknoop of korrespondensie voer oor enige nasienriglyne nie. Daar word toegegee dat verskillende menings rondom sake van beklemtoning of detail in sodanige riglyne mag voorkom. Dit is ook voor die hand liggend dat, sonder die voordeel van bywoning van 'n standaardiseringsvergadering, daar verskillende interpretasies mag wees oor die toepassing van die nasienriglyne.

MODULE 1 CALCULUS EN ALGEBRA

VRAAG 1

Laat $n = 1$ ✓

$$LK = \sum_{i=1}^1 r^{i-1} = r^0 = 1 \quad \checkmark \checkmark$$

$$RK = \frac{r^1 - 1}{r - 1} = 1 \quad \checkmark$$

∴ waar vir $n = 1$ ✓

Veronderstel ✓ waar vir $n = k$ ∴ $\sum_{i=1}^k r^{i-1} = \frac{r^k - 1}{r - 1}$ ✓

Laat $n = k + 1$ ✓

$$LK = \sum_{i=1}^{k+1} r^{i-1} = \frac{r^k - 1}{r - 1} + r^{k-1+1} \quad \checkmark \checkmark = \frac{r^k - 1 + r^{k+1} - r^k}{r - 1} \quad \checkmark$$

$$= \frac{r^{k+1} - 1}{r - 1} \quad \checkmark$$

∴ volgens die beginsel van volledige induksie ✓ is die bewering waar $\forall n \geq 1$ met $r \neq 1$ ✓

[14]

VRAAG 2

2.1 $\log_5 \frac{(x+3)(x-2)}{2} = 2$ ✓✓

$$\therefore \frac{(x+3)(x-2)}{2} = 5^2 = 25 \quad \checkmark$$

$$\therefore x^2 + x - 6 = 50 \quad \checkmark$$

$$\therefore x^2 + x - 56 = 0 \quad \checkmark$$

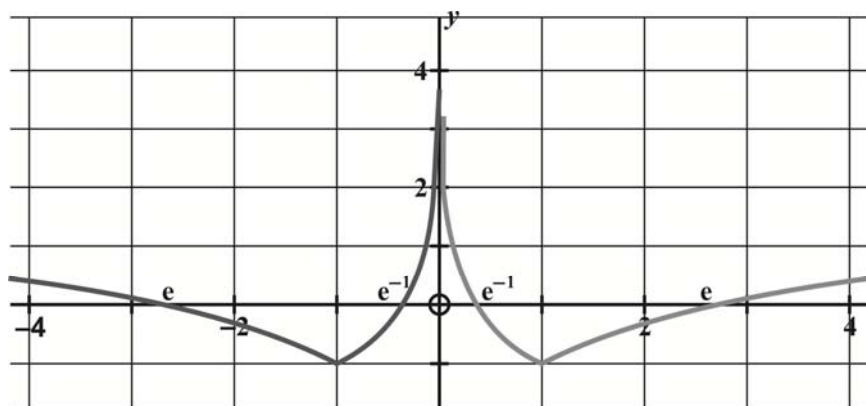
$$\therefore (x+8)(x-7) = 0 \quad \checkmark$$

$$\therefore x = -8 \text{ of } x = 7 \quad \checkmark \text{ maar } x \neq -8 \quad \checkmark$$

Dus is enigste oplossing $x = 7$. ✓

(8)

2.2



(punte: ✓ basiese vorm ✓✓ vir spitspunt by $(-1; 0)$ en ✓✓ vir spitspunt by $(1; 0)$

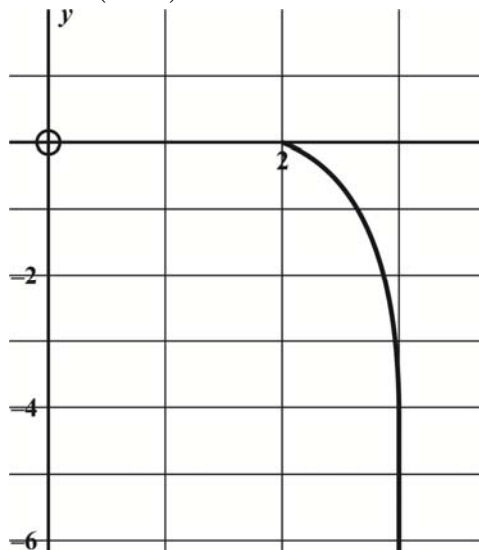
✓✓ $x = \pm e$ ✓✓ vir $x = \pm e^{-1}$ ✓ vir vertikale asimptoot by $x = 0$

(10)

2.3 (a) $g(f(x)) = \ln \left(1 - (\sqrt{x-2})^2 \right)$ ✓✓ (4)

(b) Definisiegebied: $x-2 \geq 0$ ✓ $\therefore x \geq 2$ ✓ maar ook $(1-(x-2)) > 0$ ✓ $\therefore x < 3$ ✓
Dus is die definisiegebied $2 \leq x < 3$ ✓✓ (6)

(c) Waardegebied: as $x = 2$ dan $\ln(1-0) = \ln 1 = 0$ ✓
Aangesien $x \rightarrow 3$ ✓ $\ln(3-x) \rightarrow -\infty$ ✓ (grafiek nie nodig nie, maar hieronder getoon)
Dus $g(f(x)) \leq 0$ op sy definisiegebied in $\mathbb{R}[x]$ ✓



(4)
[32]

VRAAG 3

3.1 $abi + 3bi^2$ ✓ $= (-11 - 13i)(2 - 5i)$ ✓
 $abi - 3b = -22 + 55i - 26i + 65i^2$ ✓
 $abi - 3b = -87 + 29i$ ✓
 $\therefore ab = -87$ ✓ en $-3b = 29i$ ✓
 $\therefore b = -29i$ ✓ en $a = 3$ ✓ (8)

3.2 $\sum_{n=1}^4 2i^n = 2i + 2i^2 + 2i^3 + 2i^4$
 $= 2i - 2 - 2i + 2 = 0$ ✓ (5)

3.3 $(x - (\sqrt{3} - i)) (x - (\sqrt{3} + i)) = (x - \sqrt{3})^2 - i^2$ ✓
 $= x^2 - 2\sqrt{3}x + 3 + 1$ ✓✓
 $\therefore x^2 - 2\sqrt{3}x + 4 = 0$ ✓ is 'n kwadratiese vergelyking. Enige ander toelaatbare een sal vol punte kry.

OF

$x^2 - (\sqrt{3} + i + \sqrt{3} - i)x + (\sqrt{3} - i)(\sqrt{3} + i)$
 $\therefore x^2 - 2\sqrt{3}x + 4 = 0$

(6)
[19]

VRAAG 4

$$\begin{aligned}
 4.1 \quad f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\
 &= \lim_{h \rightarrow 0} \frac{1}{h} \left(\frac{1}{\sqrt{x+h}} - \frac{1}{\sqrt{x}} \right) \\
 &= \lim_{h \rightarrow 0} \frac{1}{h} \left(\frac{\sqrt{x} - \sqrt{x+h}}{\sqrt{x}\sqrt{x+h}} \right) \times \left(\frac{\sqrt{x} + \sqrt{x+h}}{\sqrt{x} + \sqrt{x+h}} \right) \\
 &= \lim_{h \rightarrow 0} \frac{1}{h} \left(\frac{x - h - x}{(\sqrt{x}\sqrt{x+h})(\sqrt{x} + \sqrt{x+h})} \right) \\
 &= \lim_{h \rightarrow 0} \left(\frac{-1}{(\sqrt{x}\sqrt{x+h})(\sqrt{x} + \sqrt{x+h})} \right) \\
 &= \frac{-1}{2x\sqrt{x}} \quad \checkmark \quad \checkmark \text{ korrekte gebruik van limietnotasie}
 \end{aligned} \tag{8}$$

$$\begin{aligned}
 4.2 \quad (a) \quad \lim_{x \downarrow \pi} f(x) &= \pi - \pi = 0 \quad \checkmark \checkmark \\
 \lim_{x \uparrow \pi} f(x) &= \sin \pi = 0 \quad \checkmark \checkmark \\
 f(\pi) &= 0 \quad \checkmark \quad \therefore \text{kontinu by } x = \pi \quad \checkmark \\
 (b) \quad \lim_{x \downarrow \pi} f'(x) &= -1 \quad \checkmark \\
 \lim_{x \uparrow \pi} f(x) &= \cos x \quad \checkmark = -1 \quad \checkmark \\
 \text{Dus is funksie kontinu } \checkmark \text{ en die gradiënt van die linkerkant neig na dieselfde} \\
 \text{waarde as die gradient van die regterkant by } x = \pi \quad \checkmark \text{ dus is die funksie} \\
 \text{differensieerbaar by } x = \pi. \quad \checkmark
 \end{aligned} \tag{6}$$

(6)
[20]

VRAAG 5

$$\begin{aligned}
 5.1 \quad \text{Vertikale asimptoot } x &= 3 \quad \checkmark \checkmark \\
 \text{Skuins asimptoot } 2x^2 - 7x + 4 &= (x-3)(2x-1) + 1 \quad \checkmark \checkmark \\
 \therefore f(x) &= 2x-1 + \frac{1}{x-3} \quad \checkmark \checkmark \\
 \text{Dus is die skuins asimptoot } y &= 2x-1 \quad \checkmark
 \end{aligned} \tag{8}$$

$$\begin{aligned}
 5.2 \quad \text{Gebruik } f(x) &= 2x-1 + \frac{1}{x-3} \\
 \text{Dan } f'(x) &= 2 - (x-3)^{-2} \\
 \therefore \text{draaipunt: } 2 - (x-3)^{-2} &= 0 \\
 \therefore (x-3)^2 &= \frac{1}{2} \\
 \therefore x &= 3 \pm \frac{1}{\sqrt{2}} \\
 f''(x) &= \frac{2}{(x-3)^3} \quad \checkmark \text{ dus } f''\left(3 + \frac{1}{\sqrt{2}}\right) > 0 \quad \checkmark \therefore \text{min } \checkmark \text{ en } f''\left(3 - \frac{1}{\sqrt{2}}\right) < 0 \quad \checkmark \therefore \text{maks } \checkmark \\
 \text{Koördinate } (3, 71; 7, 83) &\checkmark \text{ Min en } (2, 29; 2, 17) \quad \checkmark \text{ Maks}
 \end{aligned} \tag{12}$$

5.3 y -afsnit: $f(0) = -1 - \frac{1}{3} = -\frac{4}{3}$ ✓✓

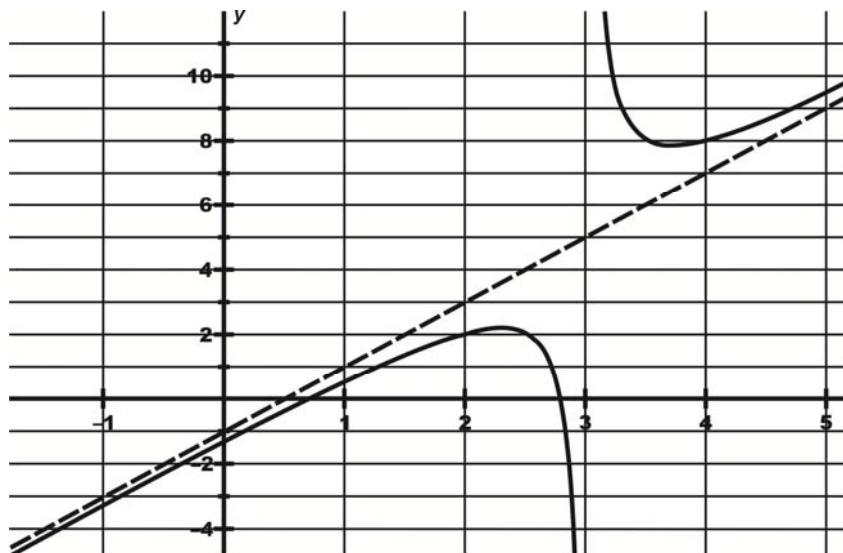
x -afsnit: $2x^2 - 7x + 4 = 0$ ✓✓

$$x = \frac{7 \pm \sqrt{17}}{4}$$
 ✓✓

(of $x = 2.78$ of $x = 0.72$)

(6)

5.4



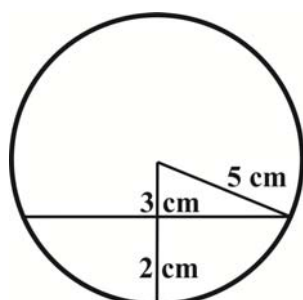
Puntetoekenning: ✓ vertikale asimptoot
 ✓ skuins asimptoot
 ✓ basiese vorm
 ✓✓✓ afsnitte met asse
 ✓✓ draaipunte

 (8)
 [34]

VRAAG 6

6.1

✓✓


 Dus 3: 4: 5 ✓ driehoek, dus is breedte $2 \times 4 = 8$ cm ✓

(4)

6.2 $\theta = \arccos\left(\frac{3}{5}\right)$ ✓ = 0.9273 ✓

$$\begin{aligned} X\text{-deursnee-oppervlakte} &= \frac{1}{2} 5^2 (2 \times 0.9273) - \frac{1}{2} 5^2 \sin(2 \times 0.9273) \\ &= 11.1825 \text{ cm}^2 \end{aligned}$$

$$\therefore \text{Volume} = 50 \times 11.1825 = 559.13 \text{ cm}^3 \text{ of (ml)}$$

 (9)
 [13]

VRAAG 7

$$7.1 \quad f'(x) = \frac{1}{\sqrt{x}} \checkmark \checkmark \sin x \checkmark + 2\sqrt{x} \checkmark \cos x \checkmark \checkmark \quad (6)$$

$$7.2 \quad \text{Vereis } h(x) = f(x) - g(x) = 0$$

$$\text{Newton Raphson-formule: } x_{n+1} = x_n \checkmark - \frac{2\sqrt{x} \sin x - x \checkmark \checkmark}{\frac{1}{\sqrt{x}} \sin x + 2\sqrt{x} \cos x - 1 \checkmark \checkmark} \text{ met } x_0 = 2.5 \checkmark \quad (6)$$

$$7.3 \quad x = 2.2848 \text{ korrek tot 4DP } \checkmark \checkmark \checkmark \checkmark \quad (4)$$

[16]

VRAAG 8

$$8.1 \quad (a) \quad 2 \int x \cos 2x dx + \int \cos 2x dx \checkmark$$

$$\text{Laat } g = x \checkmark \text{ dan } g' = 1 \checkmark \text{ en } f' = \cos 2x \checkmark \text{ dan } f = \frac{\sin 2x}{2} \checkmark$$

$$\begin{aligned} \text{Dus } 2 \int x \cos 2x dx + \int \cos 2x dx &= 2 \left(\frac{x \sin 2x}{2} \checkmark - \int \frac{\sin 2x}{2} dx \checkmark \right) + \frac{\sin 2x}{2} \checkmark + c \\ &= x \sin 2x + \frac{\cos 2x}{2} \checkmark + \frac{\sin 2x}{2} + c \checkmark \end{aligned}$$

OF

$$g = 2x + 1 \quad \therefore g' = 2$$

$$f' = \cos 2x \quad \therefore f = \frac{\sin 2x}{2}$$

$$\therefore \int (2x + 1) \cos 2x dx = (2x + 1) \cdot \frac{\sin 2x}{2} - \int 2 \cdot \frac{\sin 2x}{2} dx$$

$$= (2x + 1) \cdot \frac{\sin 2x}{2} + \frac{\cos 2x}{2} + c \quad (10)$$

$$(b) \quad \text{Laat } u = 2x^3 \checkmark \text{ dan } du = 6x^2 dx \checkmark$$

$$\text{Dus } \int x^2 \sec^2(2x^3) dx = \frac{1}{6} \checkmark \int \sec^2 u \checkmark du \checkmark = \frac{1}{6} \tan u \checkmark + c = \frac{1}{6} \tan(2x^3 \checkmark) + c \checkmark \quad (8)$$

$$(c) \quad \text{Laat } u = \cot 2x \checkmark \text{ dan } du = -\operatorname{cosec}^2 2x \cdot 2 dx \checkmark$$

$$\text{Dus } \int \cot^2 2x \operatorname{cosec}^2 2x dx = -\frac{1}{2} \checkmark \int u^2 du \checkmark$$

$$= -\frac{1}{2} \frac{u^3}{3} \checkmark + c$$

$$= -\frac{\cot^3 2x}{6} \checkmark + c \checkmark \quad (8)$$

8.2 $\text{Volume} = \int_0^1 \pi x^2 dx - \int_0^1 \pi x^4 dx \quad \checkmark \checkmark \checkmark \checkmark$

$$\left[\frac{\pi x^3}{3} \right]_{x=0}^{x=1} - \left[\frac{\pi x^5}{5} \right]_{x=0}^{x=1} \quad \checkmark \checkmark \checkmark \checkmark$$

$$\frac{\pi}{3} - \frac{\pi}{5}$$

$$= \frac{2\pi}{15} \text{ eenhede}^3 \quad \checkmark \checkmark$$

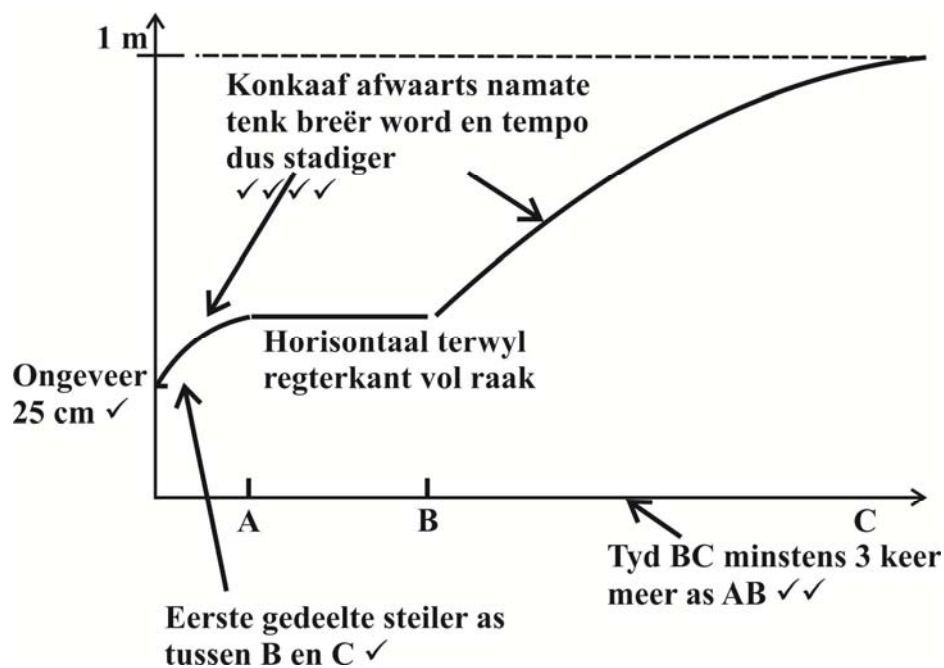
(10)

8.3 Funksie D is die integraal van Funksie A $\checkmark \checkmark \checkmark \checkmark \checkmark \checkmark$

(6)

[42]

VRAAG 9



[10]

Totaal vir Module 1: 200 punte

MODULE 2 STATISTIEK

VRAAG 1

- 1.1 Ry per motor het dalk die hoogste gemiddelde, maar die busstatistiek het 'n hoë standaardafwyking. Ewekansige steekproewe het 'n graad van veranderlikheid. (3)

- 1.2 Laat x die buspendelaars en y die motorpendelaars wees.

$$H_0 : \mu_y - \mu_x = 0$$

$$H_1 : \mu_y - \mu_x < 0$$

Verwerpingsgebied:

Verwerp H_0 as $Z < -1,75$

Toetsstatistiek:

$$z = \frac{(25,2 - 29,6)}{\sqrt{\frac{(2,8)^2}{55} + \frac{(5,2)^2}{45}}} = -5,103$$

Gevolgtrekking: Ons verwerp H_0 by die 4%-betekenispeil en stel voldoende bewys voor om die bewering te ondersteun. (10)

- 1.3 Foutgrens neem toe. Met ander woorde, jy kan 'n nulhipotese verwerp wanneer dit in werklikheid waar is. (2)
[15]

VRAAG 2

2.1 $\frac{6! \times (2!)^6}{12!} = 0,000096$ (6)

2.2 $\binom{7}{4} + 2 \times \binom{7}{3} = 105$ (6)

2.3 (a) $P(P \cap K') + P(P' \cap K) = (0,7)(0,4) + (0,6)(0,3) = 0,46$ (6)

(b) $P(P \cup K) = P(P) + P(K) - P(P \cap K)$
 $= (0,7) + (0,6) - (0,7)(0,6)$
 $= 0,88$ (6)
 [24]

VRAAG 3

3.1 (a) $r = -0,9028$

Daar is 'n sterk negatiewe lineêre korrelasie wat impliseer dat namate die ouderdom van 'n voertuig toeneem, die waarde afneem. (3)

(b) $y = 5911,2488 - 472,5744x$ (4)

(c) R591,92 (2)

(d) Betroubaar, omdat daar 'n sterk korrelasie is en 0 is nie ver buite die reikwydte nie.
OF Effens onbetroubaar, omdat 0 buite die reikwydte van die data in die tabel is. (2)

3.2 $3 = 0,482 - 0,63(7) + 0,216(x_2)$

$x_2 = \text{R}32\,000$ (3)

[14]

VRAAG 4

4.1 $\int_0^3 (kx - 3k) dx = 1$

$$\left[\frac{kx^2}{2} - 3kx \right]_0^3 = 1$$

$$\frac{9k}{2} - 9k = 1$$

$$k = \frac{-2}{9}$$

OF $\left[\frac{k(x-3)^2}{2} \right]_0^3 = 1$

$$0 - \frac{9k}{2} = 1$$

$$k = -\frac{2}{9}$$

(5)

4.2 $\frac{-2}{9} \int_0^1 (x-3) dx = \frac{5}{9}$

(3)

4.3 $\frac{-2}{9} \int_0^m (x-3) dx = 0,5$

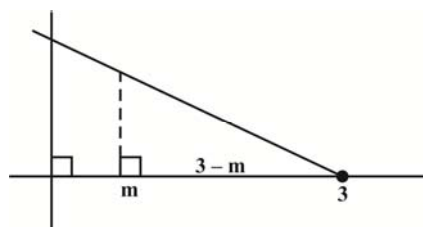
$$\left[\frac{x^2}{2} - 3x \right]_0^m = \frac{-9}{4}$$

$$\frac{m^2}{2} - 3m = \frac{-9}{4}$$

$$2m^2 - 12m + 9 = 0$$

$$m \neq 5,12 \text{ of } m = 0,8787$$

OF vir soortegelyke driehoeke



Verhouding van oppervlakte = 1 : 2

Verhouding van sye = 1 : $\sqrt{2}$

$$\therefore \frac{3-m}{3} = \frac{1}{\sqrt{2}}$$

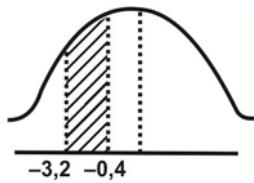
$$\therefore m = 0,8787$$

(8)

[16]

VRAAG 5

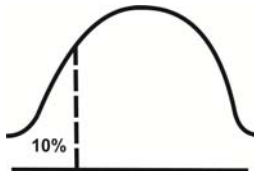
$$5.1 \quad P(156 < x < 170) = P\left(\frac{156-172}{5} < z < \frac{170-172}{5}\right)$$



$$\begin{aligned} &= P(-3,2 < z < -0,4) \\ &= 0,49931 - 0,1554 \\ &= 0,34391 \\ &\therefore 34\% \end{aligned}$$

(9)

5.2



$$\begin{aligned} -1,28 &= \frac{x-172}{5} \\ x &= 165,6 \end{aligned}$$

(6)
[15]

VRAAG 6

$$6.1 \quad P(4 \text{ opgemerk}) = \frac{50}{60} \times \frac{49}{59} \times \frac{48}{58} \times \frac{47}{57} = 0,4723$$

$$\text{OF } \frac{\binom{50}{4} \binom{10}{0}}{\binom{60}{4}} = 0,4723$$

(6)

$$6.2 \quad P(\text{minstens een}) = 1 - P(\text{geen})$$

$$\begin{aligned} &= 1 - \frac{\binom{10}{0} \binom{50}{4}}{\binom{60}{4}} \quad \text{OF} \quad 1 - 0,4723 \\ &= 0,5277 \quad \quad \quad = 0,5277 \end{aligned}$$

(4)

$$\begin{aligned} 6.3 \quad &\binom{7}{0} (0,4723)^0 (0,5277)^7 \\ &= 0,0114 \end{aligned}$$

(6)

[16]

Totaal vir Module 2: 100 punte

MODULE 3 FINANSIES EN MODELLERING

VRAAG 1

- 1.1 **R200** ✓ (1)
- 1.2 **8%** ✓✓ (2)
- 1.3 **R342,15** ✓✓ en **R169,52** ✓ (3)
- 1.4 $342,15 - 169,52$ ✓ = $172,23$ ✓
 $200 - 172,23 =$ **27,77** ✓ (3)
- 1.5 20 ✓ $\times 200 + 169,52 (1 + 0,08)$ ✓ = **4 183,08** ✓✓ (4)
- [13]**

VRAAG 2

- 2.1 $2\,500\,000 = A(1 + 0,1214)^8$ ✓✓ $A = 999\,667,55$ ✓
 $\frac{2\,500\,000}{999\,667,75}$ ✓ = $2,50083 =$ **250%** ✓ (5)
- 2.2 $F_v = \frac{50\,000 \text{ ✓ } (1 + 0,0873) \text{ ✓✓ } [(1 + 0,0873)^7 - 1] \text{ ✓✓}}{0,0873} = 496\,058,21 \text{ ✓✓}$
 $2\,500\,000 + 496\,058,21 =$ **2 996 058,21** ✓ (8)
- [13]**

VRAAG 3

$$\begin{aligned}
 3.1 \quad & \frac{10\,000 \checkmark \left[\left(1 + \frac{0,065}{4} \right)^{48} - 1 \right] \checkmark \checkmark \left(1 + \frac{0,0682}{4} \right)^{25} \checkmark \checkmark}{\frac{0,065}{4}} = 1\,095\,834,76 \checkmark \\
 & \frac{10\,000 \checkmark \left[\left(1 + \frac{0,0682}{4} \right)^{22} - 1 \right] \checkmark \checkmark \left(1 + \frac{0,0682}{4} \right)^3 \checkmark \checkmark}{\frac{0,0682}{4}} = 277\,996,32 \checkmark \\
 & 1\,095\,834,76 + 277\,996,32 = \mathbf{1\,373\,831,08} \checkmark \checkmark
 \end{aligned}$$

OF

$$\begin{aligned}
 & \frac{10\,000 \checkmark \left[\left(1 + \frac{0,065}{4} \right)^{49} - 1 \right] \checkmark \checkmark \left(1 + \frac{0,0682}{4} \right)^{24} \checkmark \checkmark}{\frac{0,065}{4}} = 1\,110\,839,05 \checkmark \\
 & \frac{10\,000 \checkmark \left[\left(1 + \frac{0,082}{4} \right)^{21} - 1 \right] \checkmark \checkmark \left(1 + \frac{0,0682}{4} \right)^3 \checkmark \checkmark}{\frac{0,0682}{4}} = 262\,992,03 \checkmark \\
 & 1\,110\,839,05 + 262\,992,03 = \mathbf{1\,373\,831,08} \checkmark \checkmark \quad (14)
 \end{aligned}$$

$$\begin{aligned}
 3.2 \quad & x \left(1 + \frac{0,0682}{4} \right)^{10} \checkmark \checkmark + x \left(1 + \frac{0,0682}{4} \right)^4 \checkmark \checkmark + x \checkmark = 1\,375\,000 \checkmark \\
 & x = \mathbf{422\,536,25} \checkmark \checkmark \quad (10) \\
 & \quad \quad \quad \mathbf{[24]}
 \end{aligned}$$

VRAAG 4

$$4.1 \quad r = \mathbf{0,103} \checkmark\checkmark \quad (2)$$

$$4.2 \quad K = -\frac{r}{m} = -\frac{0,103}{-0,000147} \checkmark\checkmark = 700,68 \checkmark \approx \mathbf{701} \checkmark \quad (4)$$

$$4.3 \quad 0,095 \checkmark\checkmark = -0,000147.P + 0,103 \quad P = 54,42 \checkmark \approx \mathbf{54} \checkmark \quad (4)$$

$$4.4 \quad P_{n+1} = \checkmark P_n + 0,103 P_n \checkmark \left(1 - \frac{P_n}{700 \checkmark}\right), \quad P_0 = 12 \checkmark$$

$$P_{15} = 49,62 \approx \mathbf{50} \checkmark\checkmark \quad (6)$$

[16]

VRAAG 5

$$5.1 \quad 24\,000 \times 5/4 \checkmark = \mathbf{30\,000} \checkmark \quad (2)$$

$$5.2 \quad \text{Doodgemaak} = 0,00056 \checkmark \times 24\,000 \times 1\,000 \checkmark = \mathbf{13\,440} \checkmark\checkmark \quad (4)$$

$$5.3 \quad 0,7 \checkmark \times 0,15 \checkmark \times 4 \times 3 \checkmark \times 0,6 \checkmark = \mathbf{0,756} \checkmark\checkmark \quad (6)$$

$$5.4 \quad 0,1116 \checkmark \times 0,00056 \checkmark \times 1\,000 \times 24\,000 \checkmark = \frac{1}{2} \checkmark\checkmark \times 1\,000 \times \text{kleintjies} \checkmark$$

$$\mathbf{Kleintjies = 3} \checkmark\checkmark \quad (8)$$

[20]

VRAAG 6

$$6.1 \quad 127 \text{ skuiwe} \checkmark\checkmark \quad (2)$$

$$6.2 \quad T_n = 2.T_{n-1} \checkmark\checkmark + 1, \checkmark T_3 = 7 \checkmark$$

OF

$$T_{n+1} = T_n + 2^n \checkmark\checkmark, T_3 = 7 \checkmark$$

OF

$$T_n = T_{n-1} \checkmark + 2^{n-1}, \checkmark\checkmark T_3 = 7 \checkmark \quad (4)$$

$$6.3 \quad 63 = a.31 + b.15 \checkmark\checkmark \quad \text{en} \quad 31 = a.15 + b.7 \checkmark\checkmark$$

$$a = 3 \checkmark \quad b = -2 \checkmark$$

$$T_n = \mathbf{3.T_{n-1} - 2.T_{n-2}} \checkmark \quad T_3 = 7 \text{ en } T_4 = \mathbf{15} \checkmark$$

OF

$$\text{Tel elke keer by 2, 4, 6, 8 ... dit is } 2^{n-1} \checkmark\checkmark$$

$$T_n = T_{n-1} \checkmark + 2.T_{n-2} \checkmark\checkmark + 2, \checkmark\checkmark T_3 = 7 \text{ en } T_4 = \mathbf{15} \checkmark \quad (8)$$

[14]

Totaal vir Module 3: 100 punte

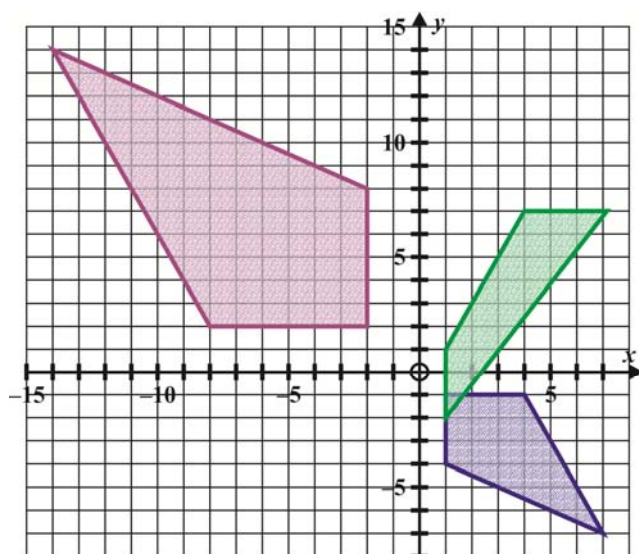
MODULE 4 MATRIKSE EN GRAFIEKTEORIE
VRAAG 1

- 1.1 (a) rotasie van 180° om die oorsprong ✓✓
 refleksie in $y = x$ ✓✓ (4)

- (b) $\begin{pmatrix} -3 & 0 \\ 0 & 1 \end{pmatrix}$ ✓ strekking
 ✓✓ faktor
 ✓ matriks (4)

- (c) $\begin{pmatrix} \cos 63,13 & -\sin 63,13 \\ \sin 63,13 & \cos 63,13 \end{pmatrix}$ ✓✓ $\begin{pmatrix} 1 \\ -4 \end{pmatrix}$ ✓✓ = $\begin{pmatrix} 4,02 \\ -0,92 \end{pmatrix}$ ✓✓ (6)

1.2



- (a) ✓✓ berekening
 ✓✓ skets (4)

- (b) ✓✓ berekening
 ✓✓ skets (4)

[22]

VRAAG 2

- 2.1 (a) 1 ✓✓ (2)

- (b) 0 ✓✓ (2)

- (c) 0 ✓✓ (2)

- 2.2 (a) k ✓✓ (2)

- (b) $-k$ ✓✓ (2)

- (c) $2a_{11} \cdot 2a_{22} \cdot 2a_{33} + 2a_{12} \cdot 2a_{23} \cdot 2a_{31} + \dots - 2a_{13} \cdot a_{22} \cdot a_{31}$ ✓✓
 $= 8[a_{11} \cdot a_{22} \cdot a_{33} + \dots - a_{13} \cdot a_{22} \cdot a_{31}]$
 $= 8k$ ✓✓ (4)

[14]

VRAAG 3

$$3.1 \quad A = \begin{pmatrix} 10 \\ 4 \\ -2 \end{pmatrix} \checkmark\checkmark \quad B = \begin{pmatrix} -4 & 7 & -1 \\ 4 & -2 & 6 \\ 12 & -6 & -2 \end{pmatrix} \checkmark\checkmark\checkmark\checkmark\checkmark\checkmark \quad C = \begin{pmatrix} -0,5 \\ 1 \\ 5 \end{pmatrix} \checkmark\checkmark\checkmark \quad (10)$$

$$3.2 \quad \begin{pmatrix} b & b & 2b \\ 0 & 0 & -2 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} b & b & 2b \\ 0 & 0 & -2 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} b & b & 2b \\ 0 & 0 & -2 \\ 0 & 0 & 1 \end{pmatrix} \checkmark\checkmark\checkmark\checkmark$$

$b = b^2$ OF $2b = 2b^2 \checkmark\checkmark$ $b = 0, \quad 1 \checkmark\checkmark$ (8)
[18]

VRAAG 4

4.1 Ja, $\checkmark$ alle nodusse van dieselfde graad. $\checkmark$ (2)

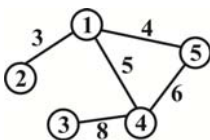
4.2 Nee, $\checkmark$ meer as 1 paar onewe nodusse. $\checkmark$ (2)

4.3 A B $\checkmark$ F $\checkmark$ G $\checkmark$ H $\checkmark$ E $\checkmark$ HD $\checkmark$ C $\checkmark$ BA $\checkmark$ = 60 $\checkmark\checkmark$ (10)

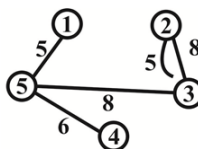
4.4 A E H G F B C D A = 49
 $\checkmark\checkmark$ 8 nodusse
 $\checkmark\checkmark$ kringbaan
 $\checkmark\checkmark\checkmark\checkmark$ gordellengte ≤ 50
 Verskeie opsies moontlik. (8)
 [22]

VRAAG 5

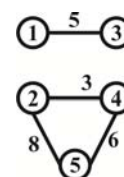
5.1 $\checkmark\checkmark\checkmark\checkmark$ sy kante korrek verbind
 $\checkmark\checkmark$ sy kantgewig (6)



5.2 H: $\checkmark\checkmark$ matriks is nie simmetries nie $\checkmark\checkmark$ OF 'n skets (4)



5.3 G: $\checkmark\checkmark$ $G_1 - G_3$ nie met ander sy kante verbind nie $\checkmark\checkmark$ OF 'n skets (4)
 [14]

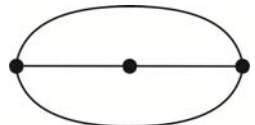


VRAAG 6

6.1 (a) $n - 1 \checkmark\checkmark$ (2)

(b) $2n - 2 \checkmark\checkmark$ (2)

(c) $(n - 1)/2 \checkmark\checkmark$ (2)

6.2  $\checkmark$ 3 nodusse
 $\checkmark\checkmark$ 4 sy kante
 $\checkmark$ skets (4)
 [10]

Totaal vir Module 4: 100 punte

Totaal: 300 punte