



GRAAD 12-EKSAMEN
NOVEMBER 2014

GEVORDERDEPROGRAM-WISKUNDE

NASIENRIGLYNE

Tyd: 3 uur

300 punte

Hierdie nasienriglyne word voorberei vir gebruik deur eksaminatore en hulp-eksaminatore. Daar word van alle nasieners vereis om 'n standaardiseringsvergadering by te woon om te verseker dat die nasienriglyne konsekwent vertolk en toegepas word tydens die bepunting van kandidate se skrifte.

Die IEB sal geen gesprek aanknoop of korrespondensie voer oor enige nasienriglyne nie. Daar word toegegee dat verskillende menings rondom sake van beklemtoning of detail in sodanige riglyne mag voorkom. Dit is ook voor die hand liggend dat, sonder die voordeel van bywoning van 'n standaardiseringsvergadering, daar verskillende interpretasies mag wees oor die toepassing van die nasienriglyne.

MODULE 1 CALCULUS EN ALGEBRA

VRAAG 1

Stap 1: Laat $n = 1$

$$LK = 1^2 = RK$$

Stap 2: Neem aan waar vir $n = k$

$$\text{dan } 1 + 3 + 5 + \dots + (2k - 1) = k^2$$

Stap 3: Vir $n = k + 1$

$$1 + 3 + 5 + \dots + (2k - 1) + (2k + 1)$$

$$= k^2 + 2k + 1$$

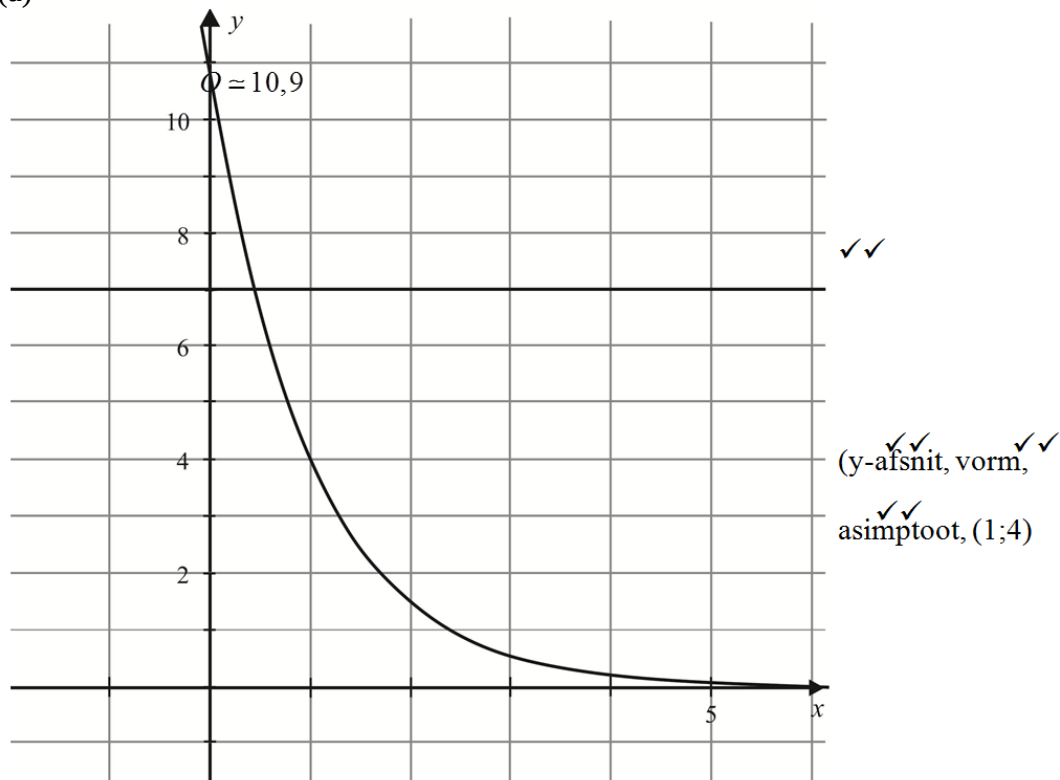
$$= (k + 1)^2$$

$\therefore$ volgens die bewys van volledige induksie is die stelling waar vir $n \in \mathbb{N}$.

[12]

VRAAG 2

2.1 (a)



(8)

(b)

$$4e^{1-x} > 7$$

$$e^{1-x} > 1.75$$

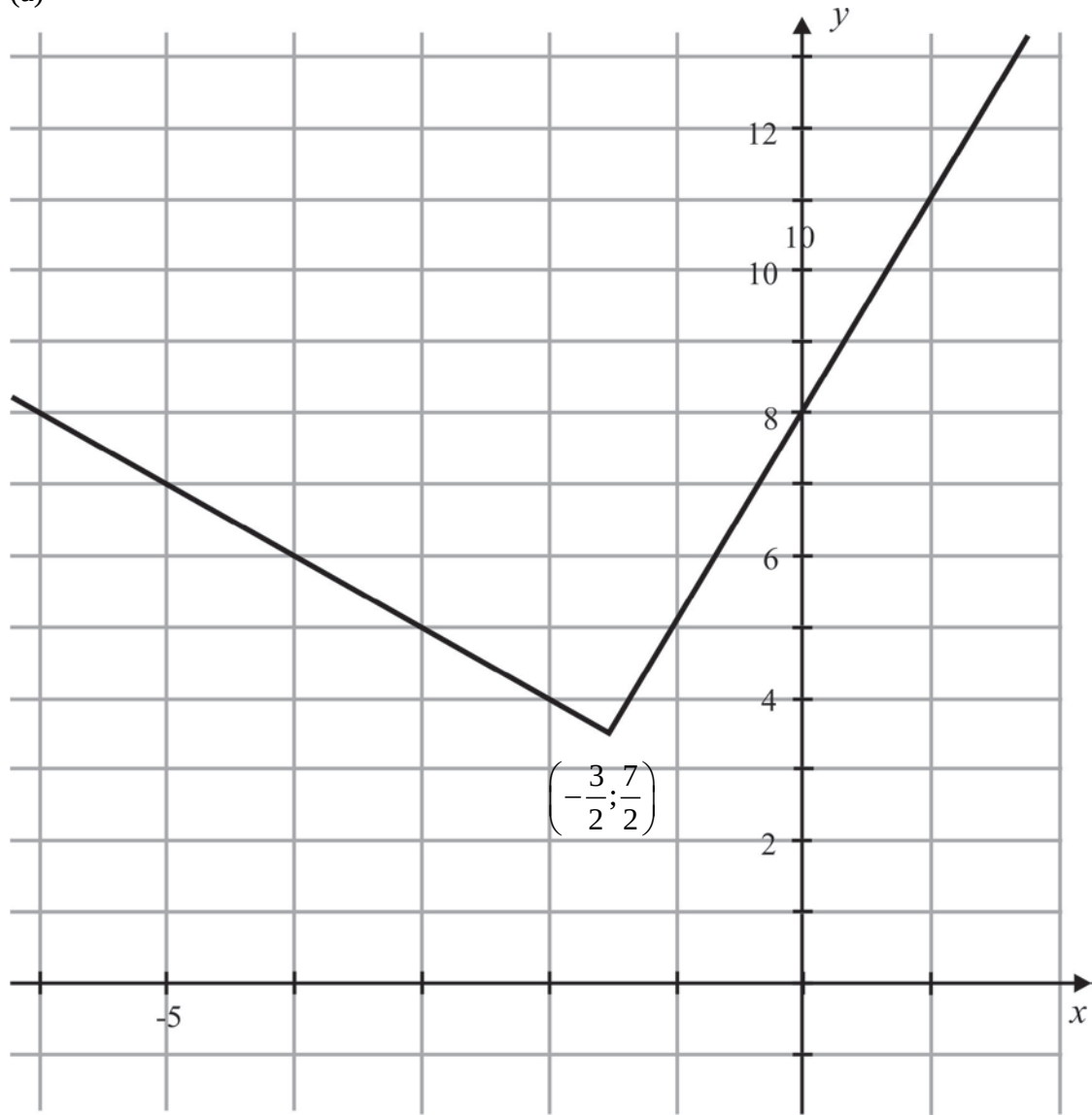
$$\ln 1.75 < 1 - x$$

$$0.56 < 1 - x$$

$$x < 0.44$$

(6)

2.2 (a)



(8)

(b) Vir $g(x) \neq f(x)$ vereis $-1 < k \leq 3$

(4)

[26]

VRAAG 3

$$\begin{aligned}
 3.1 \quad & 2(5-2i) - i(6i-1) \\
 & = 10 - 4i - 6i^2 + i \\
 & = 16 - 3i
 \end{aligned} \tag{6}$$

$$\begin{aligned}
 3.2 \quad & \frac{x-10}{(2x-1)(x+3)} = \frac{A}{2x-1} + \frac{B}{x+3} = \frac{Ax+3A+2Bx-B}{(2x-1)(x+3)} \\
 & \therefore x: A+2B=1 \\
 & \text{Konstante: } +3A-B=-10 \\
 & \therefore A = -\frac{19}{7} \quad B = \frac{13}{7} \quad \text{dus } \frac{x-10}{(2x-1)(x+3)} = \frac{-\frac{19}{7}}{2x-1} + \frac{\frac{13}{7}}{x+3}
 \end{aligned} \tag{10}$$

$$\begin{aligned}
 3.3 \quad & \lim_{n \rightarrow \infty} \frac{4\sum k - 3\sum 1}{n^2} = \lim_{n \rightarrow \infty} \frac{1}{n^2} \left(\frac{4n(n+1)}{2} - 3n \right) \\
 & = \lim_{n \rightarrow \infty} \left(2 - \frac{1}{n} \right) = 2
 \end{aligned} \tag{8}$$

[24]

VRAAG 4

$$\begin{aligned}
 4.1 \quad & \text{Aangesien } f'(x) > 0 \text{ is } f \text{ altyd toenemend} \\
 (a) \quad & \text{Grootste waarde van } f(x) \text{ is by } x = F
 \end{aligned} \tag{4}$$

$$\begin{aligned}
 (b) \quad & f''(x) = 0 \therefore \text{ buigpunte by (en } f(x) \neq 0) \\
 & x = B; C; E
 \end{aligned} \tag{6}$$

$$\begin{aligned}
 4.2 \quad & \text{Nee. Ons het geen aanduiding van wat die } y\text{-waardes vir die grafiek is nie (geen} \\
 & \text{gegewe punte).}
 \end{aligned} \tag{4}$$

$$\begin{aligned}
 4.3 \quad & f'(x) \neq 0 \quad \text{dus het } f(x) \text{ geen draaipunte nie.}
 \end{aligned} \tag{4}$$

[18]

VRAAG 5

$$5.1 \quad y = (\sin 2x)(2-x)^{-3}$$

$$\therefore \frac{dy}{dx} = 2 \cos 2x (2-x)^{-3} + 3(2-x)^{-4} \cdot \sin 2x$$

$$\text{OF} \quad y = \frac{\sin 2x}{(2-x)^3}$$

$$\therefore \frac{dy}{dx} = \frac{2 \cos 2x (2-x)^3 + 3(2-x)^2 \cdot \sin 2x}{(2-x)^6} \quad (6)$$

$$5.2 \quad (a) \quad 1^3 + 2^3 - 1 \times 2^2 = 1 + 8 - 4 = 5$$

$$\therefore \text{punt } (1; 2) \text{ lê op kromme.} \quad (2)$$

$$(b) \quad 3x^2 + 3y^2 \frac{dy}{dx} - \left(1 \cdot y^2 + 2y \frac{dy}{dx} \cdot x \right) = 0$$

$$3x^2 + 3y^2 \frac{dy}{dx} - y^2 - 2xy \frac{dy}{dx} = 0$$

$$\therefore \frac{dy}{dx} (3y^2 - 2xy) = y^2 - 3x^2$$

$$\therefore \frac{dy}{dx} = \frac{y^2 - 3x^2}{3y^2 - 2xy} \quad (10)$$

$$(c) \quad \text{By } (1; 2) \quad \frac{dy}{dx} = \frac{4-3}{12-4} = \frac{1}{8}$$

$$\therefore y = \frac{1}{8}x + c$$

$$2 = \frac{1}{8} + c$$

$$c = 1\frac{7}{8}$$

$$\therefore \text{raaklyn } y = \frac{1}{8}x + 1\frac{7}{8} \quad (4)$$

$$\begin{aligned}
 5.3 \quad (a) \quad & y = (bx + c)^{-1} \\
 & \therefore \frac{dy}{dx} = -1(bx + c)^{-2} \cdot b = -b(bx + c)^{-2} \\
 & \frac{d^2y}{dx^2} = 2b(bx + c)^{-3} \cdot b = 2b^2(bx + c)^{-3} \\
 & \frac{d^3y}{dx^3} = -6b^2(bx + c)^{-4} \cdot b = -6b^3(bx + c)^{-4} \quad (6)
 \end{aligned}$$

$$(b) \quad \frac{d^n y}{dx^n} = (-1)^n n! b^n (bx + c)^{-(n+1)} \quad (6)$$

[34]

VRAAG 6

$$6.1 \quad SA = 2rh + r\alpha h + 2 \times \frac{1}{2} r^2 \alpha \quad \text{maar } r = h \quad \therefore A = 2r^2 + r^2 \alpha + r^2 \alpha$$

$$= 2r^2(\alpha + 1) \quad (8)$$

$$\begin{aligned}
 6.2 \quad & Vol = \frac{1}{2} r^2 \alpha h \\
 & \text{Indien } r = h \text{ en totale volume } 1\,000 \text{ cm}^3 \\
 & \text{Dan } 1000 = \frac{1}{2} r^3 \alpha \\
 & \text{Dus } \alpha = \frac{2000}{r^3} \tag{4}
 \end{aligned}$$

$$\begin{aligned} 6.3 \quad & \therefore A = 2r^2 \left(1 + \frac{2000}{r^3} \right) = 2r^2 + \frac{4000}{r} \\ & \therefore \frac{dSA}{dr} = 4r - \frac{4000}{r^2} = 0 \quad \text{vir minimum buiteoppervlakte (SA)} \\ & \therefore 4r^3 - 4000 = 0 \\ & \therefore r = 10 \quad \text{en dus } \alpha = 2 \end{aligned} \tag{8}$$

VRAAG 7

7.1 (a) Vertikale asimptote: $(x+5)(x+1) = 0$
 $\therefore x = -5$ of $x = -1$ (4)

(b) $2x^3 + 6x^2 - 25x - 39 = (x^2 + 6x + 5)(2x - 6) + x - 9$
 $\therefore \frac{2x^3 + 6x^2 - 25x - 39}{x^2 + 6x + 5} = 2x - 6 + \frac{x - 9}{x^2 + 6x + 5}$
 $\therefore$ skuins asimptoot $= y = 2x - 6$ (8)

(c) $f''(x) > 0 =$ Impliseer konkaaf op
 $\therefore -5 < x < -1$ (4)

7.2 (a) $f(x) = \cos x - 0,25x$ aanvanklike beraming
 $f'(x) = -\sin x - 0,25$
 $\therefore x_{n+1} = x_n + \frac{\cos x - 0,25x}{\sin x + 0,25}$
 $x = -2,1333$ (8)

(b) Indien $f'(x) = 0$ d.w.s. $\sin x = -0,25$
 of $x = \sin^{-1}\left(-\frac{1}{4}\right)$
 Algebraïes deling deur 0
 OF Grafies sal nie x -as sny nie (2)
[26]

VRAAG 8

8.1 Laat $u = 3x$ $\therefore \frac{du}{3} = dx$
 $\therefore \frac{1}{3} \int \cos^2 u du = \frac{1}{3} \int \frac{1}{2} (1 + \cos 2u) du$
 $= \frac{1}{6} \left(u + \frac{\sin 2u}{2} \right) + c$
 $= \frac{x}{2} + \frac{\sin 6x}{12} + c$ (8)

$$\begin{aligned}
 8.2 \quad & \int \cos 2\theta \sin 5\theta d\theta \\
 &= \frac{1}{2} \int \sin(5\theta + 2\theta) + \sin(5\theta - 2\theta) d\theta \\
 &= \frac{1}{2} \int \sin 7\theta + \sin 3\theta d\theta \\
 &= \frac{1}{2} \left[\frac{-\cos 7\theta}{7} - \frac{\cos 3\theta}{3} \right] + c \\
 &= \frac{-\cos 7\theta}{14} - \frac{\cos 3\theta}{6} + c
 \end{aligned} \tag{6}$$

$$\begin{aligned}
 8.3 \quad & \text{Laat } u = 2 - x \quad \text{dan } x = 2 - u \\
 & du = -dx \\
 & \therefore -\int \frac{10 - 5u}{u^{\frac{1}{2}}} du = -\int 10u^{-\frac{1}{2}} - 5u^{\frac{1}{2}} du \\
 &= \frac{-10u^{\frac{1}{2}}}{\frac{1}{2}} - \frac{5u^{\frac{3}{2}}}{\frac{3}{2}} + c \\
 &= 20u^{\frac{1}{2}} - \frac{10}{3}u^{\frac{3}{2}} + c \\
 &= -20(2 - x)^{\frac{1}{2}} - \frac{10}{3}(2 - x)^{\frac{3}{2}} + c
 \end{aligned} \tag{8}$$

[22]

VRAAG 9

$$9.1 \quad \therefore V = \pi \int_0^4 \left[\frac{(x-4)^2}{4} \right]^2 dx = \frac{\pi}{16} \int_0^4 (x-4)^4 dx = \frac{\pi}{16} \left[\frac{(x-4)^5}{5} \right]_0^4 = \frac{\pi}{16} \times \frac{1024}{5} = \frac{64}{5} \pi \text{ eenhede}^3 \tag{8}$$

9.2 Nee. Die lyn $y = x$, nie 'n simmetrie-as nie $\therefore$ rotasie sal verskillend wees. Of enige ander rede. (4)

$$\begin{aligned}
 9.3 \quad & \pi \int_0^a \frac{(x+4)^4}{16} dx = \frac{\pi}{16} \left[\frac{(x-4)^5}{5} \right]_0^a = 10\pi \\
 &= \frac{\pi}{80} \left[(a-4)^5 - (-4)^5 \right] = 10\pi \\
 &\therefore (a-4)^5 = 800 - 1024 \\
 &\therefore a - 4 = -2,9515 \\
 &\therefore a = 1,0485
 \end{aligned} \tag{6}$$

[18]

Totaal vir Module 1: 200 punte

MODULE 2 STATISTIEK

VRAAG 1

1.1 (a) $X \sim N(38,4 ; 4,6^2)$

$$P(30 < x < 40) = P\left(\frac{30 - 38,4}{4,6} < z < \frac{40 - 38,4}{4,6}\right)$$

$$= P(-1,83 < z < 0,35)$$

$$= 0,4664 + 0,1368$$

$$= 0,6032 \quad (8)$$

(b) $P(x > k) = 0,9$

$$z = -1,28$$

$$-1,28 = \frac{k - 38,4}{4,6}$$

$$k = 32,512 \quad (8)$$

1.2 (a) $2,33 \left(\sqrt{\frac{(0,32)(0,68)}{n}} \right) \leq 0,1$

$$\sqrt{\frac{0,2176}{n}} \leq 0,0429$$

$$118 \leq n \quad (6)$$

(b) 'n 98%-vertrouensinterval vir p is

$$0,32 \pm 2,33 \left(\sqrt{\frac{(0,32)(0,68)}{118}} \right)$$

$$(0,2199 ; 0,4201) \quad (4)$$

[26]

VRAAG 2

- 2.1 (a) Onwaar
 (b) Waar
 (c) Onwaar
 (d) Waar
 (e) Waar
 (f) Onwaar (6)

$$2.2 \quad H_0 : \mu = 8$$

$$H_1 : \mu < 8$$

Verwerpingsgebied:

Verwerp H_0 indien $z < -1,75$

Toetsstatistiek:

$$z = \frac{7,92 - 8}{\frac{0,2}{\sqrt{30}}} = -2,19$$

Gevolgtrekking:

Aangesien $z < -1,75$, verwerp ons H_0 by die 4%-betekenispeil en stel voldoende bewys voor om die bewering te staaf.

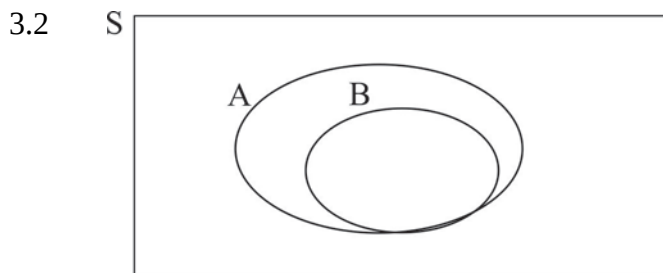
(10)

[16]

VRAAG 3

3.1 (a) A en B is onafhanklik. (2)

(b) $P(A \cap B) = 0 \therefore$ A en B is onderling uitsluitend. (2)



(4)

$$3.3 \quad \frac{\binom{4}{4} \binom{4}{0}}{\binom{8}{4}} = \frac{1}{70} \quad \text{of} \quad \frac{4}{8} \times \frac{3}{7} \times \frac{2}{6} \times \frac{1}{5} = \frac{1}{70} \quad (8)$$

$$3.4 \quad P(x > 4) = \binom{6}{5} (0,6)^5 (0,4)^1 + \binom{6}{6} (0,6)^6 (0,4)^0 = 0,2333 \quad (8)$$

$$3.5 \quad \binom{5}{4} + \binom{5}{3} + \binom{5}{2} + \binom{5}{1} = 30 \quad (8)$$

[32]

VRAAG 4

4.1 (a)

x	1^1	2	3
$P(x)$	$\frac{7}{24}$	$\frac{8}{24}$	$\frac{9}{24}$

$$\frac{7}{24} + \frac{8}{24} + \frac{9}{24} = 1$$

$\therefore P(X = x)$ is 'n waarskynlikheidsmassafunksie. (4)

$$\begin{aligned} \text{(b)} \quad \bar{x} &= 1\left(\frac{7}{24}\right) + 2\left(\frac{8}{24}\right) + 3\left(\frac{9}{24}\right) \\ &= \frac{25}{12} \end{aligned} \quad (5)$$

$$\begin{aligned} 4.2 \quad P(\text{korrek}) &= 1(0,65) + \frac{1}{5}(0,35) \\ &= 0,72 \end{aligned} \quad (4)$$

$$\begin{aligned} P(\text{raaiskote/korrek}) &= \frac{\frac{1}{5}(0,35)}{0,65 + \frac{1}{5}(0,35)} \\ &= \frac{7}{72} = 0,0972 \end{aligned} \quad (4)$$

[17]

VRAAG 5

5.1 (a) Vervang $(21; \bar{y})$ in $y = -7x + 163$

$$\begin{aligned} \bar{y} &= -7(21) + 163 \\ &= 16 \end{aligned} \quad (2)$$

$$\begin{aligned} \text{(b)} \quad y &= -7(5) + 163 \\ &= 128 \end{aligned} \quad (2)$$

$$5.2 \quad r = -\frac{1}{7} = -0,1429$$

'n Korrelasiekoëffisiënt is negatief en word as baie swak beskou. (3)

5.3 Die beraming is onbetroubaar, aangesien die korrelasie te swak is. (2)

[9]

Totaal vir Module 2: 100 punte

MODULE 3 FINANSIES EN MODELLERING

VRAAG 1

$$1.1 \quad T_{n+1} = T_n \left(1 + \frac{0,09}{4} \right) - 11\,300, \quad T_0 = 150\,000 \quad (5)$$

$$1.2 \quad 0,5 = (1-c)^4 \quad \therefore c = 15,91\% \\ 0,1 = (1-0,1591)^n \quad \therefore n = 13,3 \quad \therefore n = 14 \text{ yearsjaar} \quad (9)$$

[14]

VRAAG 2

$$2.1 \quad OB = \frac{8932,75 \left[1 - \left(1 + \frac{0,0648}{12} \right)^{-51} \right]}{\frac{0,0648}{12}} = 397\,290,48 \quad (6)$$

$$2.2 \quad 400\,000 \left(1 + \frac{0,0674}{12} \right)^3 = \frac{x \left[1 - \left(1 + \frac{0,0674}{12} \right)^{-48} \right]}{\frac{0,0674}{12}} \\ x = 9\,691,81 \quad (8)$$

$$2.3 \quad 8\,932,75 \times 189 + 9\,691,81 \times 48 - 1\,200\,000 = 953\,449,38 \quad (6)$$

[20]

VRAAG 3

$$3.1 \quad \left(1 + \frac{0,058}{4} \right)^4 = \left(1 + \frac{i}{12} \right)^{12} \\ 1 + \frac{i}{12} = 1,004\,810 \dots \\ i = 5,7722\% \quad \text{per jaar, maandeliks saamgestel} \quad (8)$$

$$3.2 \quad 850\,000 = \frac{7\,300 \left[1 - \left(1 + \frac{0,057722}{12} \right)^{-n} \right]}{\frac{0,057722}{12}} \quad n = 171,128$$

$171,128 \div 12 = 14,261 \approx 14 \text{ jaar, 4 maande}$
Hy sal verband **8 maande** gouer afbetaal.

(10)
[18]

VRAAG 4

$$\begin{array}{ll} 4.1 & \begin{array}{ll} 0,95 = 1 + 0,6 - x & x = \mathbf{0,65} \\ 500 = -2\,500 + y & y = \mathbf{3\,000} \end{array} \end{array} \quad (6)$$

$$4.2 \quad F_{n+1} = 0,95 \cdot F_n + 500, \quad F_0 = 5\,000 \quad F_{20} = \mathbf{8\,207/8}$$

Populasie neem toe, maar teen stadiger tempo met verloop van tyd. (5)

$$4.3 \quad F_n = 0,95^{100} \cdot 5000 + \frac{500[0,95^{100} - 1]}{0,95 - 1} = \mathbf{9\,970} \quad (5)$$

[16]

VRAAG 5

$$5.1 \quad (a) \quad \begin{array}{ll} \text{prooi } 20\,000 & \text{roofdier } 1\,000 \end{array} \quad (2)$$

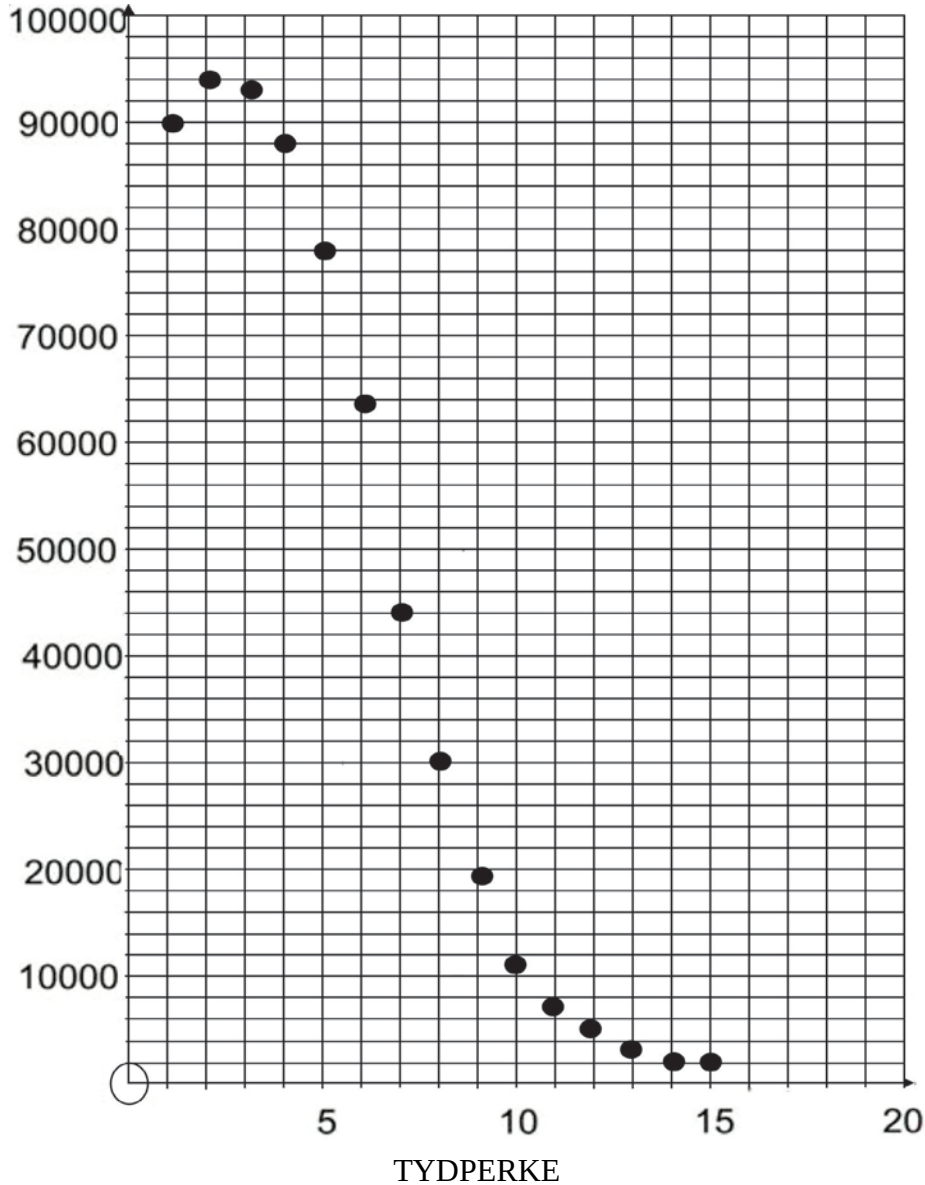
$$(b) \quad 1\,000 < \text{prooi} < 94\,000 \quad (\text{aanvaar } 500 - 2\,000; 93\,000 - 94\,000) \quad (2)$$

$$(c) \quad \text{kwadrant } 3 \quad (2)$$

$$(d) \quad \text{koers van dodelike wisselwerkings tussen roofdier en prooi} \quad (2)$$

$$(e) \quad \begin{array}{l} \text{Kan lei tot uitsterwing van prooi} \\ \text{Prooi sal vinniger verminder, en in sommige tydperke is prooipopulasie} \\ \text{reeds baie laag (1\,000).} \end{array} \quad (4)$$

POPULASIES



(8)
[20]

VRAAG 6

6.1 $Q_{n+1} = 0,12P_n + 0,9Q_n + 400\,000$ (4)

6.2 Vir ewewig $P_{n+1} = P_n$: $-0,2P + 0,06Q = -600\,000$.
 Vir ewewig $Q_{n+1} = Q_n$: $0,12P - 0,1Q = -400\,000$.
 $P = 6\,562\,500$, $Q = 11\,875\,000$ (8)
 [12]

Totaal vir Module 3: 100 punte

MODULE 4 MATRIKSE EN GRAFIEKTEORIE

VRAAG 1

$$1.1 \quad (a) \quad 4a - (-2) = 0 \quad \mathbf{a} = -1/2 \quad (4)$$

$$(b) \quad \frac{1}{6} \begin{pmatrix} 4 & 1 \\ 2 & 2 \end{pmatrix} \quad (4)$$

$$\begin{array}{ll} \text{(c)} & a - 4 = -7 & \mathbf{a = -3} \\ & 4 - 2b = 4a = -12 & \mathbf{b = 8} \end{array} \quad (6)$$

$$1.2 \quad (a) \quad q \times p \quad (2)$$

$$(b) \quad p \times r \quad (2)$$

$$(c) \quad \mathbf{r} \times \mathbf{r} \quad (2)$$

[20]

VRAAG 2

$$2.1 \quad (a) \quad \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} \quad (2)$$

$$(b) \quad \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} \quad \begin{pmatrix} 1 & 0 \\ 3 & 1 \end{pmatrix} = \begin{pmatrix} 0 & -1/3 \\ 3 & 1 \end{pmatrix} \quad (6)$$

$$2.2 \quad 360^\circ \div 8 = 45^\circ$$

$$\begin{pmatrix} \cos 45 & -\sin 45 \\ \sin 45 & \cos 45 \end{pmatrix} \begin{pmatrix} -1 \\ 2 \end{pmatrix} = \begin{pmatrix} -2,121 \\ 0,707 \end{pmatrix} \quad (6)$$

$$2.3 \quad \begin{pmatrix} \cos 2A & \sin 2A \\ \sin 2A & -\cos 2A \end{pmatrix} = \begin{pmatrix} -0,8 & -0,6 \\ -0,6 & 0,8 \end{pmatrix}$$

$$\begin{aligned}\cos 2A &= -0,8 \\ 2A &= 180^\circ + 36,87^\circ = 216,87^\circ \\ \tan A &= -3\end{aligned}$$

$$\begin{aligned} \text{EN} \quad & \sin 2A = -0,6 \\ & \text{dus } A = 108,435^\circ \\ & \mathbf{y = -3x} \end{aligned} \tag{10}$$

VRAAG 3

3.1 $\det = 16$ (2)

$$3.1 \begin{pmatrix} 4 & 2 & 6 \\ -1 & 2 & 2 \\ 0 & 1 & 3 \end{pmatrix} \quad (2)$$

3.2 $a = 2 \times 2 - 2 \times 6 = -8$

$$b = 4 \times 3 - 0 \times 6 = 12$$

$$c = 4 \times 2 - (-1) \times 2 = 10 \quad (6)$$

[10]

VRAAG 4

4.1 Dijkstra (2)

4.2 Prim (2)

4.3 Fleury (2)

4.4 Naaste Buurpunt (2)

[8]

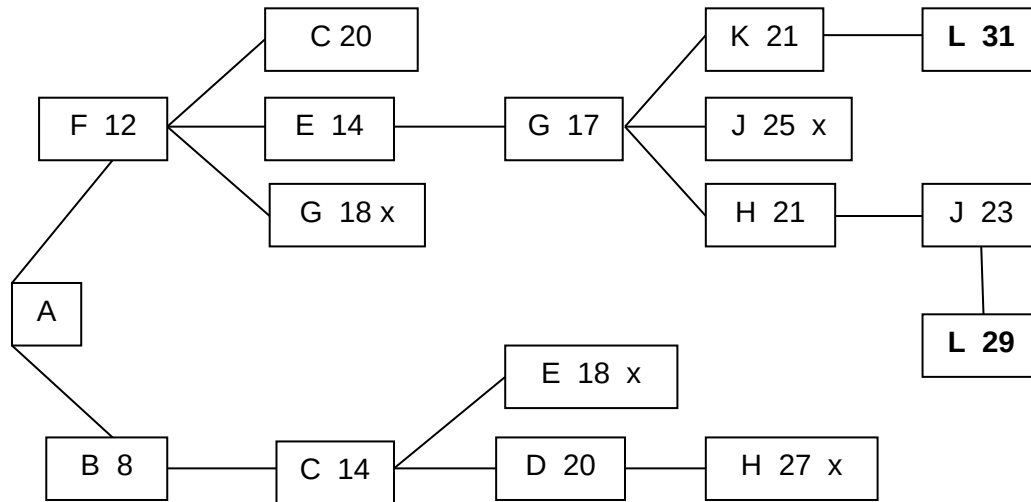
VRAAG 5

5.1

	A	B	C	D	E	F	G	H	J	K	L
A											
B	8										
C		6									
D			6								
E			4								
F	12		8		2						
G					3	6					
H				7			4				
J							8	2			
K							4				
L									6	10	

(6)

5.2



aanduiding van 'n opgeskorte AB-roete
besonderhede van roete via AF

A F E G H J L = 29

(12)

5.3 EG HJ

(4)

5.4 (a) 11 nodusse, dus moet elke nodus met 5 ander nodusse verbind word.
Slegs waar van G.

(2)

(b) Omgekeerde nie waar nie

(2)

[26]

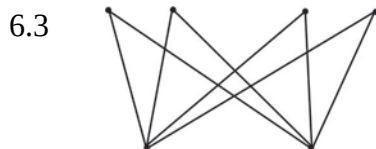
VRAAG 6

6.1 (a) $m + n$

(2)

(b) $m.n$ (2)

6.2 11 of 7 (4)



6.3

nodusse
skakels

(4)

[12]

Totaal vir Module 4: 100 punte

Totaal: 300 punte