



GRAAD 12-EKSAMEN
NOVEMBER 2015

**GEVORDERDEPROGRAM-WISKUNDE
KERNMODULE: CALCULUS EN ALGEBRA**

NASIENRIGLYNE

Tyd: 2 uur

200 punte

Hierdie nasienriglyne is opgestel vir gebruik deur eksaminators en sub-eksaminators van wie verwag word om almal 'n standaardiseringsvergadering by te woon om te verseker dat die riglyne konsekwent vertolk en toegepas word by die nasien van kandidate se skrifte.

Die IEB sal geen bespreking of korrespondensie oor enige nasienriglyne voer nie. Ons erken dat daar verskillende standpunte oor sommige aangeleenthede van beklemtoning of detail in die riglyne kan wees. Ons erken ook dat daar sonder die voordeel van die bywoning van 'n standaardiseringsvergadering verskillende vertolkings van die toepassing van die nasienriglyne kan wees.

VRAAG 1

1.1 (a) $e^{\pi x} = 3$
 $\therefore \pi x = \ln 3$
 $\therefore x = \frac{\ln 3}{\pi} = 0,35$ (4)

(b) $\tan x = 2$
 $\therefore x = 1,107 + \pi k$
 $\tan x = -2$
 $\therefore x = -1,107 + \pi k$
 $\therefore x = 1,11$ of $2,04$ of $4,25$ of $5,18$ (6)

(c) $\log_{\frac{1}{2}} x + \log_2 x^n = -2$ $\therefore \log_{\frac{1}{2}} \frac{1}{2} + \log_2 \left(\frac{1}{2}\right)^n = -2$
 $\therefore -\log_2 x + n \log_2 x = -2$ $\therefore 1 + \log_2 2^{-n} = -2$
 $\therefore -\log_2 \frac{1}{2} + n \log_2 \frac{1}{2} = -2$ **OF** $1 - n \log_2 2 = -2$
 $\therefore 1 - n = -2$ $1 - n = -2$
 $\therefore n = 3$ $n = 3$ (4)

1.2 (a) (1) $P = 70 \times 1 + 22$
 $= 92^\circ\text{C}$ (2)

(2) Limiet namate $t \rightarrow \infty$
 $P = 22^\circ\text{C}$ (2)

(b) $55 = 70 \times 1,2^{-t} + 22$ $40 = 70 \times 1,2^{-t} + 22$
 $t = 4,1245 \text{ min}$ $t = 7,449 \text{ min}$
 $4,12 \leq t \leq 7,45$ (7)
[25]

VRAAG 2

Bewys waar vir $n = 1$:

$$LK = 5^2 - 1 = 24$$

wat 'n veelvoud is van 8.

Neem aan waar vir $n = k$: $5^{2k} - 1 = 8p$ $p \in N$

Bewys waar vir $n = k + 1$:

$$5^{2(k+1)} - 1$$

$$= 5^2 \cdot 5^{2k} - 1$$

Maar $5^{2k} = 8p + 1$ uit aanname

$$= 25(8p + 1) - 1$$

$$= 200p + 24$$

$$= 8(25p + 3) \text{ wat 'n veelvoud is van 8}$$

Gevolgtik het ons getoon dat as die uitdrukking deelbaar is deur 8 vir enige een natuurlike waarde van n , dan is dit ook waar vir die volgende opeenvolgende waarde. Maar dit is waar vir $n = 1$, derhalwe ook waar vir $n = 2, 3, 4$ en so verder vir alle natuurlike waardes van n .

OF

$\therefore$ volgens die bewys van volledige induksie is die bewering waar vir $n \in N$.

[14]

VRAAG 3

$$\begin{aligned} 3.1 \quad & \frac{(a+bi)(a+bi)}{a^2+b^2} \\ &= \frac{a^2-b^2+2abi}{a^2+b^2} \end{aligned}$$

$$\therefore \text{ reële deel } = \frac{a^2-b^2}{a^2+b^2} \quad (7)$$

3.2 Een ander oplossing is $x = 3 + 7i$.
 $x - 3 = 7i$

$$\therefore x^2 - 6x + 9 = -49$$

$$\therefore x^2 - 6x + 58 = 0$$

Met inspeksie: $(2x-1)(x^2-6x+58) = 2x^3 + px^2 + qx - 58$

$$p = -13 \quad q = 122$$

(10)

[17]

VRAAG 4

$$\begin{array}{ll} 4.1 & \tan \theta = \sqrt{3} & r^2 = 1 + 3 \text{ (Pythag)} \\ & \therefore \theta = \frac{\pi}{3} \text{ radiale} & \therefore r = 2 \end{array} \quad (6)$$

$$\begin{array}{ll} 4.2 & \text{Oppervlakte van sektor} = \\ & \frac{1}{2} r^2 \theta \\ & = \frac{1}{2} (2)^2 \cdot \frac{\pi}{3} \\ & = 2,09 \text{ eenhede}^2 \end{array} \quad \begin{array}{l} \tan \theta = \frac{AB}{2} \\ \therefore AB = 3,46 \text{ eenhede} \end{array}$$

Oppervlakte van $\triangle OAB$

$$\begin{array}{l} \frac{1}{2} \times 2 \times 3,46 \\ = 3,46 \text{ eenhede}^2 \end{array}$$

$$\text{Gearseerde oppervlakte} = 3,46 - 2,09 = 1,37 \text{ eenhede}^2 \quad (9)$$

[15]

VRAAG 5

$$\begin{aligned}
 5.1 \quad (a) \quad p(x) &= (2+3x)^{-1} \\
 \therefore p'(x) &= -(2+3x)^{-2} (3) \\
 &= -\frac{3}{(2+3x)^2} \quad (4)
 \end{aligned}$$

$$\begin{aligned}
 (b) \quad x &= \frac{1}{2+3y} \\
 \therefore 3y &= \frac{1}{x} - 2 \\
 \therefore p^{-1}(x) &= \frac{1-2x}{3x} \quad (4)
 \end{aligned}$$

$$5.2 \quad (a) \quad p(q(x)) \quad (3)$$

$$(b) \quad r(p(x)) \quad (3)$$

$$\begin{aligned}
 5.3 \quad g(x) &= \frac{x^2+6}{2x-5} \\
 \therefore g'(x) &= \frac{(2x-5)(2x) - (x^2+6)(2)}{(2x-5)^2} \\
 \therefore 0 &= 4x^2 - 10x - 2x^2 - 12 \\
 \therefore 0 &= x^2 - 5x - 6 \\
 \therefore x &= 6; \quad x = -1 \\
 y &= 6; \quad y = -1 \quad (8) \\
 & \quad [22]
 \end{aligned}$$

VRAAG 6

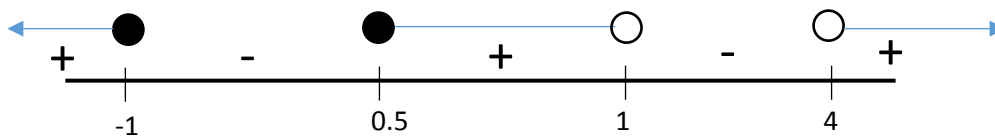
- (a) (i) Daar sal twee x -afsnitte wees, tensy 'n faktor kanselleer, d.w.s.

$$(x + 1) \text{ gee } p = 5. \quad (2x - 1) \text{ gee } p = -8,5$$

- (ii) $p = 5$ en $p = -8,5$ (sien bo)

Noemer is 'n volkome vierkant, d.w.s. $x = -4$ of $x = 4$.

(b) (i)
$$f(x) = \frac{2x^2 + x - 1}{x^2 - 5x + 4} = \frac{(2x-1)(x+1)}{(x-4)(x-1)}$$



$$x \geq -1 \text{ of } 0,5 \leq x < 1 \text{ of } x > 4$$



x -afsnitte: $x = -1$; $x = 0.5$

y -afsnit: $y = -0.25$

vertikale asimptote: $x = 1$ en $x = 4$

horisontale asimptoot: $y = 2$

vorm

6.2 (a) Bewys kontinuïteit:

$$\lim_{x \rightarrow 0^-} g(x)$$

$$= 3$$

$$\lim_{x \rightarrow 0^+} g(x)$$

$$= |-3| = 3$$

$$g(3) = |-3| = 3$$

$\therefore$ kontinu by $x = 0$

Bewys differensieerbaarheid:

$$\lim_{x \rightarrow 0^-} g'(x)$$

$$\lim_{x \rightarrow 0^-} (-2x - 1)$$

$$= -1$$

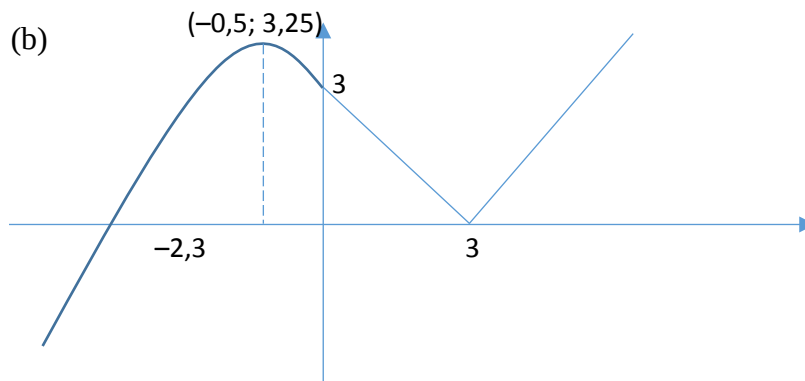
$$\lim_{x \rightarrow 0^+} g'(x)$$

$$\lim_{x \rightarrow 0^+} -1$$

$$= -1$$

$\therefore$ differensieerbaar by $x = 0$

(10)



(8)
[42]

VRAAG 7

$$7.1 \quad (a) \quad f(x) = 3(x-2)^2 - 1 - \frac{4}{x}$$

$$f(2) = -3$$

$$f(3) = \frac{2}{3}$$

Verandering van teken impliseer oplossing lê in interval, met inagneming van kontinuïteit op die interval $x \in (2;3)$.

(3)

$$(b) \quad f'(x) = 6(x-2) + \frac{4}{x^2}$$

$$x_{r+1} = x_r - \frac{3(x-2)^2 - 1 - \frac{4}{x}}{6(x-2) + \frac{4}{x^2}}$$

$$x_1 = 2,5$$

$$x_2 = 3,008241758...$$

$$x_3 = 2,897330865...$$

$$x_4 = 2,891354053...$$

$$x_n = 2,891337 \quad (6dp)$$

$$= 0,87$$

(7)

$$7.2 \quad (a) \quad \frac{d}{dx} \sin y = \frac{d}{dx} x$$

$$\therefore \cos y \cdot \frac{dy}{dx} = 1$$

$$\therefore \frac{dy}{dx} = \frac{1}{\cos y}$$

$$= \frac{1}{\sqrt{1-x^2}}$$

(5)

$$(b) \quad \sin y = 0,5$$

$$y = \frac{\pi}{6} \text{ radiale}$$

$$\therefore \frac{dy}{dx} = \frac{1}{\cos y}$$

$$= \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3} = 0,87$$

(4)

[19]

VRAAG 8

$$8.1 \quad 36 \text{ eenhede}^2 \quad (2)$$

$$8.2 \quad A_6 = 36 + \frac{48}{16} + \frac{16}{16^2} \quad \% \text{ fout} = \frac{39,0625 - 36}{36} \times 100$$

$$= 39,0625 \quad = 8,5\% \quad (4)$$

$$8.3 \quad \int_0^4 qx^2 + 1 dx = 36$$

$$\therefore \left[\frac{qx^3}{3} + x \right]_{x=0}^{x=4} = 36$$

$$\therefore \frac{64q}{3} + 4 = 36$$

$$\therefore q = \frac{3}{2} \quad (7)$$

[13]

VRAAG 9

$$9.1 \quad \int x(2x-1)^{\frac{2}{3}} dx = x \cdot \frac{(2x-1)^{\frac{5}{3}}}{\frac{5}{3} \times 2} - \frac{3}{10} \int (2x-1)^{\frac{5}{3}} \cdot 1 dx$$

$$= \frac{3x(2x-1)^{\frac{5}{3}}}{10} - \frac{3}{10} \times \frac{(2x-1)^{\frac{8}{3}}}{\frac{8}{3} \times 2}$$

$$= \frac{3x(2x-1)^{\frac{5}{3}}}{10} - \frac{9(2x-1)^{\frac{8}{3}}}{160} + C \quad (8)$$

$$9.2 \quad 2 \int \frac{1}{2} x^{-\frac{1}{2}} (\sqrt{x} + 1)^3 dx$$

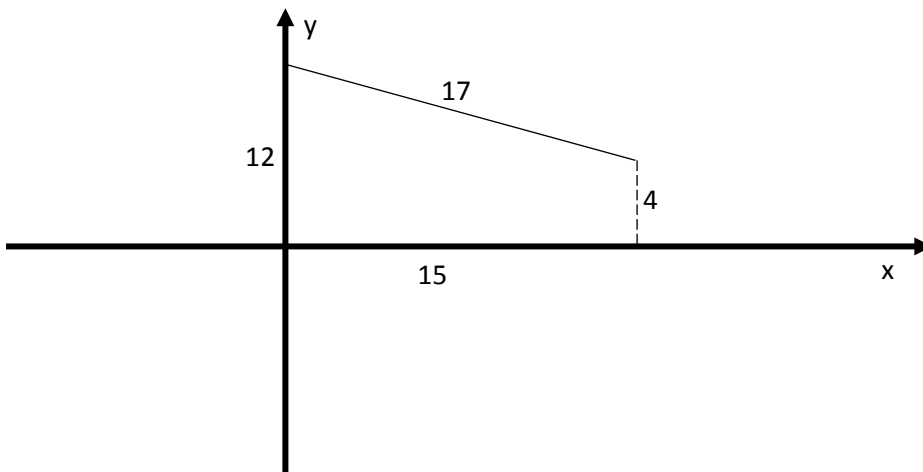
$$= 2 \frac{(\sqrt{x} + 1)^4}{4} + C$$

$$= \frac{(\sqrt{x} + 1)^4}{2} + C \quad (7)$$

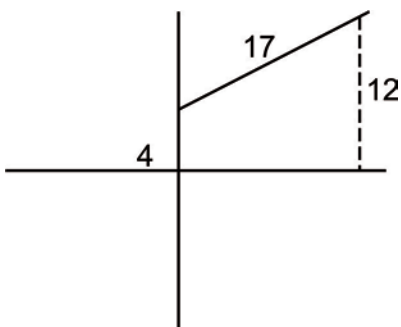
9.3 (a) $\sec^2 x - 2 \sec x \tan x + \tan^2 x$
 $= \sec^2 x - 2 \sec x \tan x + \sec^2 x - 1$
 $= 2 \sec^2 x - 2 \sec x \tan x - 1$
 $a = 2, b = -1, c = -1$ (5)

(b) $\int 2 \sec^2 x - 2 \sec x \tan x - 1 \, dx$
 $= 2 \tan x - 2 \sec x - x + C$ (5)
[25]

VRAAG 10



OF



$$V = \pi \int_0^{15} \left(12 - \frac{8}{15}x \right)^2 dx$$

Of enige ekwivalent.

[8]

Totaal: 200 punte