

**GEVORDERDEPROGRAM-WISKUNDE: VRAESTEL II**

**NASIENRIGLYNE**

Tyd: 1 uur

100 punte

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Hierdie nasienriglyne word voorberei vir gebruik deur eksaminatore en hulpeksaminatore. Daar word van alle nasieners vereis om 'n standaardiseringsvergadering by te woon om te verseker dat die nasienriglyne konsekwent vertolk en toegepas word tydens die nasien van kandidate se skrifte.

Die IEB sal geen gesprek aanknoop of korrespondensie voer oor enige nasienriglyne nie. Daar word toegegee dat verskillende menings rondom sake van beklemtoning of detail in sodanige riglyne mag voorkom. Dit is ook voor die hand liggend dat, sonder die voordeel van bywoning van 'n standaardiseringsvergadering, daar verskillende interpretasies mag wees oor die toepassing van die nasienriglyne.

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**MODULE 2 STATISTIEK**

**VRAAG 1**

$$1.1 \quad (a) \quad P(X = 3) = \binom{10}{3} \left(\frac{1}{5}\right)^3 \left(\frac{4}{5}\right)^7$$

$$= 0,2013 \quad (5)$$

$$(b) \quad 15 \left(\frac{1}{5}\right) = 3 \quad (2)$$

$$(c) \quad P(x \geq 1) > 0,95$$

$$1 - \binom{n}{0} \left(\frac{1}{5}\right)^0 \left(\frac{4}{5}\right)^n > 0,95$$

$$\left(\frac{4}{5}\right)^n < 0,05$$

$$n \log \frac{4}{5} < \log 0,05$$

$$n > 13,4251$$

$$14 \text{ mense} \quad (8)$$

$$1.2 \quad (a) \quad 3! \times 4! \times 8! \times 3! = 34\,836\,480 \quad (5)$$

$$(b) \quad \frac{\binom{3}{2} \binom{4}{2} \binom{8}{2}}{\binom{15}{6}} = \frac{72}{715} \text{ of } 0,1007 \quad (5)$$

$$(c) \quad \binom{8}{3} \binom{3}{1} \binom{4}{2} + \binom{8}{3} \binom{3}{2} \binom{4}{1} + \binom{8}{4} \binom{3}{1} \binom{4}{1}$$

$$= 2\,520 \quad (8)$$

**[33]**

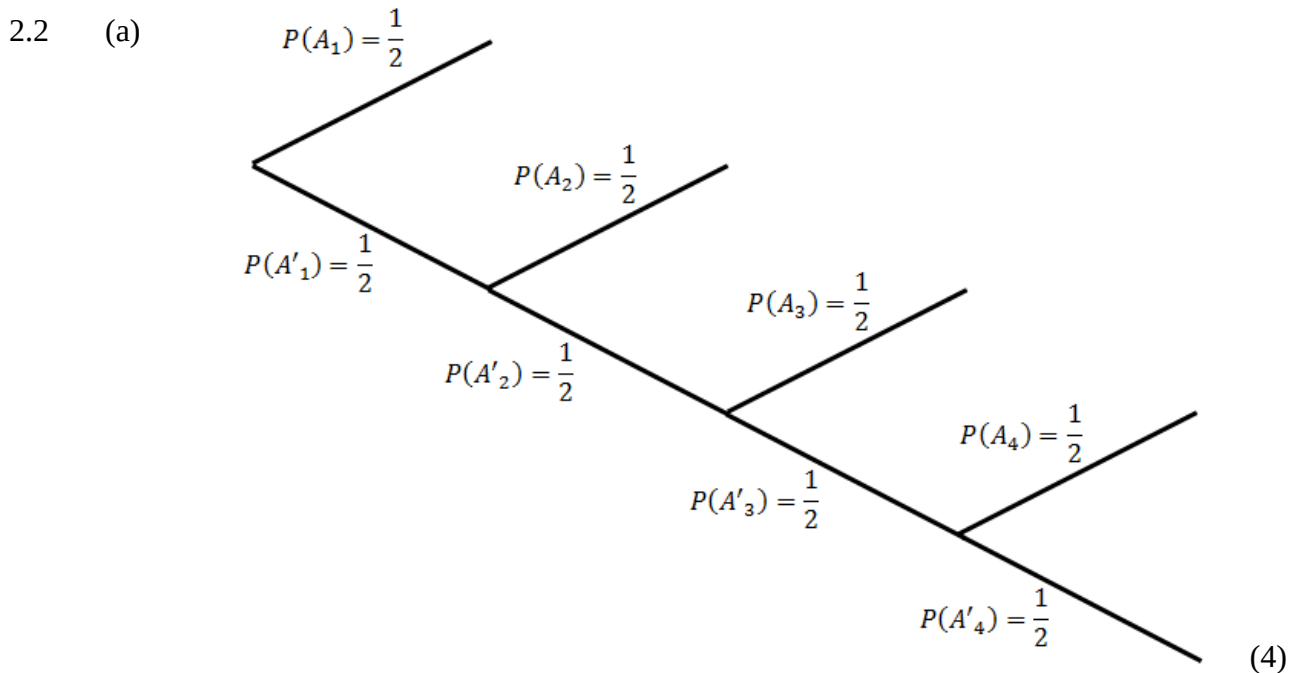
**VRAAG 2**

2.1 (a)  $y = 10,38 - 1,54x$  (4)

(b)  $r = -0,983$   
Data lê naby aan die regressielyn. (2)

(c)  $y = 10,38 - 1,54(35)$   
 $= -43,52$  (2)

(d) Betroubaar maar versigtigheid is nodig want alhoewel ekstrapolasie voorkom, is korrelasie baie sterk. (2)



(b)

|            |               |               |               |                |                |
|------------|---------------|---------------|---------------|----------------|----------------|
| $x$        | 0             | 1             | 2             | 3              | 4              |
| $P(X = x)$ | $\frac{1}{2}$ | $\frac{1}{4}$ | $\frac{1}{8}$ | $\frac{1}{16}$ | $\frac{1}{16}$ |

(4)

(c)  $E[X] = 0\left(\frac{1}{2}\right) + 1\left(\frac{1}{4}\right) + 2\left(\frac{1}{8}\right) + 3\left(\frac{1}{16}\right) + 4\left(\frac{1}{16}\right) = 0,9375$  (2)

**[20]**

**VRAAG 3**

3.1 (a) 'n 96%-vertrouensinterval vir  $p$  is:

$$0,8 \pm 2,05 \sqrt{\frac{(0,8)(0,2)}{250}}$$

$$(0,7481 ; 0,8519) \quad (6)$$

(b) 96% van die tyd sal die interval die populasieproporsie bevat. (2)

3.2 (a)  $\sim N(\mu ; \sigma^2)$

$$\sigma = \frac{3}{5} \mu$$

$$P(x < 2\mu) = P\left(z < \frac{2\mu - \mu}{\frac{3}{5}\mu}\right)$$

$$= P(z < 1,67)$$

$$= 0,5 + 0,4525$$

$$= 0,9525 \quad (8)$$

(b)  $-0,84 = \frac{\frac{1}{3}\mu - \mu}{2}$

$$\frac{-42}{52} = -0,84 \times 2 = \frac{-2}{3} \mu$$

$$\mu = +2,52$$

(7)  
**[23]**

### VRAAG 4

4.1 'n Eenkantige toets moet gebruik word aangesien Gareth graag wil sien of daar 'n toename in sy reistyd is van verlede jaar af na hierdie jaar. (2)

4.2  $H_0 : \mu = 45,7$

$H_1 : \mu > 45,7$

Verwerpingsgebied: verwerp  $H_0$  indien  $z > 1,75$

Toetsstatistiek:

$$z = \frac{47,4 - 45,7}{\frac{3,2}{\sqrt{8}}} = 1,5026$$

Gevolgtrekking: aangesien  $z < 1,75$ , verwerp ons nie die  $H_0$  by die 4%-betekenispeil nie en stel onvoldoende bewys voor om die bewering te staaf, gevolglik geen betekenisvolle toename in Gareth se reistyd nie. (10)

[12]

### VRAAG 5

5.1

|                | 1             | 2             | 3             | 4             |
|----------------|---------------|---------------|---------------|---------------|
| <b>Arnie</b>   | $\frac{1}{2}$ | $\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{1}{6}$ |
| <b>Michael</b> | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{1}{4}$ |
| <b>Connor</b>  | $\frac{1}{7}$ | $\frac{1}{7}$ | $\frac{2}{7}$ | $\frac{3}{7}$ |

Vir Ingang 1:

$$\begin{aligned} P(x \geq 2) &= P(A \cap M \cap C') + P(A \cap M' \cap C) + P(A' \cap M \cap C) + P(A \cap M \cap C) \\ &= \left(\frac{1}{2}\right)\left(\frac{1}{4}\right)\left(\frac{6}{7}\right) + \left(\frac{1}{2}\right)\left(\frac{3}{4}\right)\left(\frac{1}{7}\right) + \left(\frac{1}{2}\right)\left(\frac{1}{4}\right)\left(\frac{1}{7}\right) + \left(\frac{1}{2}\right)\left(\frac{1}{4}\right)\left(\frac{1}{7}\right) \\ &= \frac{11}{56} \end{aligned} \quad (6)$$

5.2

$$\begin{aligned} &\left(\frac{1}{2}\right)\left(\frac{1}{4}\right)\left(\frac{1}{7}\right) + \left(\frac{1}{6}\right)\left(\frac{1}{4}\right)\left(\frac{1}{7}\right) + \left(\frac{1}{6}\right)\left(\frac{1}{4}\right)\left(\frac{2}{7}\right) + \left(\frac{1}{6}\right)\left(\frac{1}{4}\right)\left(\frac{3}{7}\right) \\ &= \frac{3}{56} \end{aligned} \quad (6)$$

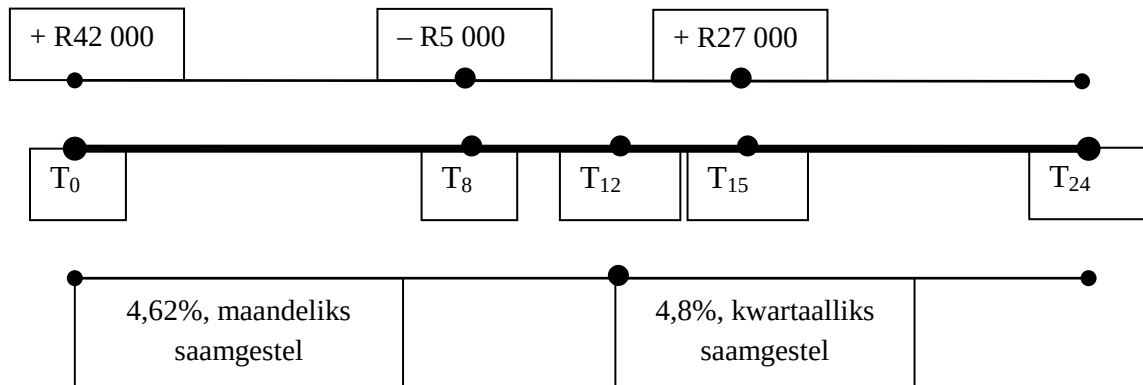
[12]

**Totaal vir Module 2: 100 punte**

### MODULE 3 FINANSIES EN MODELLERING

#### VRAAG 1

1.1



(6)

$$1.2 \quad 42\,000 \left(1 + \frac{0,0462}{12}\right)^{12} \left(1 + \frac{0,048}{4}\right)^4 = 46\,131,4622$$

$$5\,000 \left(1 + \frac{0,0462}{12}\right)^4 \left(1 + \frac{0,048}{4}\right)^4 = 5\,325,58533$$

$$27\,000 \left(1 + \frac{0,048}{4}\right)^3 = 27\,983,71066$$

$$46\,131,46 - 5\,325,58 + 27\,983,71 = \mathbf{68\,789,59}$$

OF

$$42\,000 \left(1 + \frac{0,0462}{12}\right)^8 = 43\,311,1661$$

$$38\,311,1661 \left(1 + \frac{0,0462}{12}\right)^4 = 38\,904,5740$$

$$38\,904,5740 \left(1 + \frac{0,048}{4}\right) = 39\,371,4289$$

$$66\,371,4289 \left(1 + \frac{0,048}{4}\right)^3 = \mathbf{68\,789,59} \quad (10)$$

[16]

## VRAAG 2

$$1\,000\,000 - 30\,000 = \frac{30\,000 \left[ 1 - \left( 1 + \frac{0,064}{4} \right)^{-n} \right]}{\frac{0,064}{4}}$$

$$0,482\,666 = 1,016^{-n}$$

$$n = 45,890 \text{ kwartale} = \mathbf{11 \text{ jaar, 5 of 6 maande}}$$

OF

$$1\,000\,000 = \frac{30\,000 \left( 1 + \frac{0,064}{4} \right)^n \left[ 1 - \left( 1 + \frac{0,064}{4} \right)^{-(n+1)} \right]}{\frac{0,064}{4}}$$

$$0,475\,066 = 1,016^{-(n+1)}$$

$$\therefore n = 45,890 \text{ kwartale} = \mathbf{11 \text{ jaar, 5 of 6 maande}}$$

[12]

## VRAAG 3

$$\begin{array}{ll} 3.1 & 33\,000 = 30\,400(1+r) & r = 8,55\% \\ & 30\,400 = 28\,000(1+r) & r = 8,57\% \end{array} \quad (6)$$

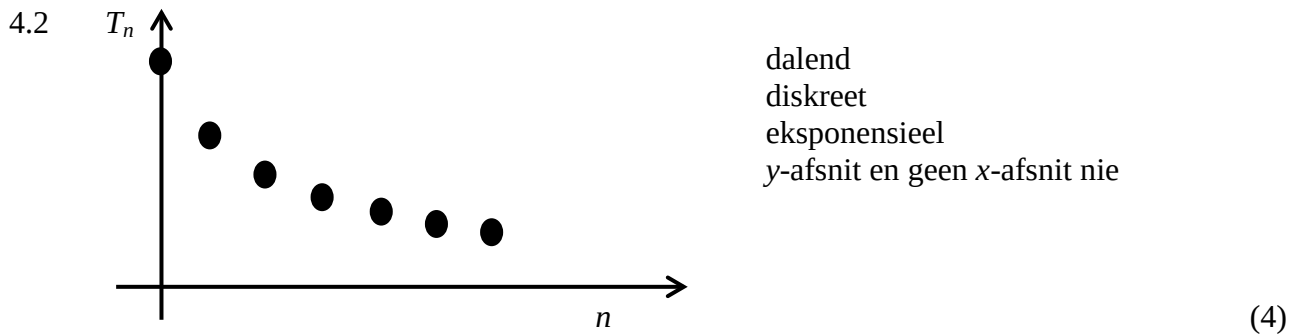
$$3.2 \quad \frac{270 \left[ \left( 1 + \frac{0,052}{12} \right)^{18 \times 12 + 1} - 1 \right]}{\frac{0,052}{12}} = \mathbf{96\,928,20} \quad (6)$$

$$3.3 \quad 96\,928,20 - 270 \times (18 \times 12 + 1) = \mathbf{38\,338,20} \quad (4)$$

$$\begin{array}{l} 3.4 \quad 33\,000 + 30\,400 \left( 1 + \frac{0,052}{12} \right)^{12} + 28\,000 \left( 1 + \frac{0,052}{12} \right)^{24} \\ \quad = \mathbf{96\,080,86} \\ \quad \mathbf{Ja, dit sal die koste dek.} \end{array} \quad \begin{array}{l} (8) \\ [24] \end{array}$$

## VRAAG 4

$$\begin{array}{l} 4.1 \quad T_3 = 4(2) + 3(-2) - 4(-1)^{2-1} = 6 \\ \quad T_4 = 4(-2) + 3(2) - 4(-1)^{3-1} = -6 \\ \quad T_5 = 4(2) + 3(-2) - 4(-1)^{4-1} = 6 \end{array} \quad (5)$$



4.3

$$P_{n+1} = \widehat{54 + r \cdot 54} \left(1 - \frac{54}{120}\right) = 54 + 29,7r$$

$$P_{n+2} = P_{n+1} + r \cdot P_{n+1} \left(1 - \frac{P_{n+1}}{120}\right)$$

$$70 = (54 + 29,7r) + r(54 + 29,7r) \left(1 - \frac{54 + 29,7r}{120}\right)$$

$$r = 0.27 \text{ ("solve" funksie)}$$

(9)  
[18]

### VRAAG 5

- 5.1 Enkeldatapunte, nie met 'n kromme verbind nie.  
Metings by intervalle geneem; metings tussen intervalle nie oorweeg nie. (2)
- 5.2  $1\,800 < \text{prooi} < 3\,800$  (2)
- 5.3 35–40 (2)
- 5.4 koers van afname : steiler gradiënt (2)
- 5.5 (a) b – koers van dodelike interaksies tussen roofdier en prooi (3)
- (b)  $1/c$  = lewensduur van roofdier; gevolglik leef roofdiere vir korter tydperke.  
OF  $cF$  = hoeveel roofdiere in elke siklus vrek; gevolglik vrek meer roofdiere in elke siklus.  
Minder roofdiere impliseer toename in prooipopulasies of stadiger afname. (3)
- 5.6 C Prooi neem aanvanklik toe en roofdier neem aanvanklik af met neiging na ewewig. (4)  
[18]

### VRAAG 6

- 6.1 G (2)
- 6.2 31 (2)
- 6.3  $T_{n+1} = 2 \cdot T_n + 1, T_1 = 1$  (4)
- 6.4  $1\,000\,000 = 2^n - 1$   $n < 19,93 = 19$  (4)  
[12]

**Totaal vir Module 3: 100 punte**

## MODULE 4                      MATRIKSE EN GRAFIEKTEORIE

### VRAAG 1

$$1.1 \quad \begin{array}{l} 2 \times 4 + 4 \times (-1) = z \\ x + 2y = 4 \quad \text{en} \quad 4x - y = -11 \end{array} \quad \begin{array}{l} z = 4 \\ x = -2, y = 3 \end{array} \quad (8)$$

1.2 (a) onwaar (2)

(b) onwaar (2)

(c) waar (2)

(d) waar (2)

**[16]**

### VRAAG 2

2.1 (a) verplasing 3 eenhede na regs en 1 eenheid af (3)

(b) ja (1)

$$(c) \quad M = \begin{pmatrix} 4 & 2 & 5 \\ 3 & 2 & -1 \end{pmatrix} \quad (2)$$

2.2 (a) dwarsdruk van faktor 3, y-as invariant (3)

(b) nee (1)

(c) determinant (1)

(d)  $\det = 1$  oppervlakte van figuur en beeld onveranderd (3)

$$2.3 \quad \begin{array}{l} \tan A = 3 \quad A = 71,565^\circ \\ \begin{pmatrix} \cos 143,13 & \sin 143,13 \\ \sin 143,13 & -\cos 143,13 \end{pmatrix} \begin{pmatrix} \cos 30 & -\sin 30 \\ \sin 30 & \cos 30 \end{pmatrix} = \begin{pmatrix} -0,39 & 0,92 \\ 0,92 & 0,39 \end{pmatrix} \end{array} \quad (8)$$

**[22]**

### VRAAG 3

$$3.1 \quad \begin{pmatrix} 6 & 2 & -3 \\ 6 & 3 & 1 \\ 9 & 3 & -2 \end{pmatrix}^{-1} = \frac{1}{15} \begin{pmatrix} -9 & -5 & 11 \\ 21 & 15 & -24 \\ -9 & 0 & 6 \end{pmatrix} \text{ OF } \begin{pmatrix} -0,6 & -0,333 & 0,733 \\ 1,4 & 1 & -1,6 \\ -0,6 & 0 & 0,4 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{15} \begin{pmatrix} -9 & -5 & 11 \\ 21 & 15 & -24 \\ -9 & 0 & 6 \end{pmatrix} \begin{pmatrix} 2 \\ 15 \\ 8 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{15} \begin{pmatrix} -5 \\ 75 \\ 30 \end{pmatrix} = \begin{pmatrix} -\frac{1}{3} \\ 5 \\ 2 \end{pmatrix} \quad (8)$$

$$3.2 \quad \begin{aligned} 6(3t - 3) - 2(6t - 9) - 3(18 - 27) &= 0 \\ 6t + 27 &= 0 \end{aligned} \quad t = -4,5 \quad (6)$$

$$3.3 \quad \text{skaalfaktor van 1,5:} \quad w = 3 \quad (2)$$

**[16]**

### VRAAG 4

$$4.1 \quad VW \quad (2)$$

$$4.2 \quad \text{Driehoekongelykheid geld nie.} \quad (2)$$

$$4.3 \quad W \ V \ R \ Q \ U \ P \ S \ T \ W \quad \text{of} \quad W \ V \ R \ U \ Q \ P \ S \ T \ W \quad (4)$$

$$4.4 \quad \begin{aligned} PSTW &= 15 \\ 2 + 3 + x + 3 + 2 &< 15 \end{aligned} \quad x < 5 \quad \text{dus } x = 4 \quad (6)$$

**[14]**

## VRAAG 5

5.1 Engels, Swahili (2)

5.2  $E \rightarrow X (48) \rightarrow Z (45) \rightarrow S (72) \rightarrow F (64) \rightarrow P (58) \rightarrow A (60) \rightarrow E (85)$   
= **432 min** (8)

5.3  $E \rightarrow P (65) \rightarrow A (60) \rightarrow F (62) \rightarrow S (64) \rightarrow X (62) \rightarrow Z (45) \rightarrow E (50)$   
= **408 min**

**OF**

$E \rightarrow Z (50) \rightarrow X (45) \rightarrow S (62) \rightarrow F (64) \rightarrow A (62) \rightarrow P (60) \rightarrow E (65)$   
= **408 min**

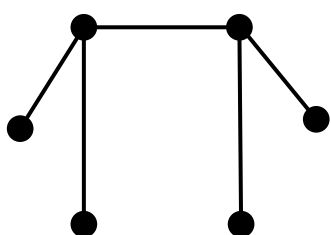
**OF enige ander aanvaarbare kring < 412 min** (10)  
[20]

## VRAAG 6

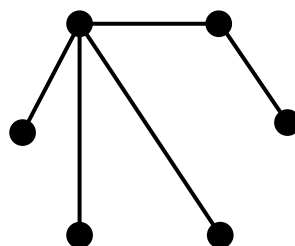
6.1 5 skakels (2)

6.2  $2 \times 5 = 10$  (2)

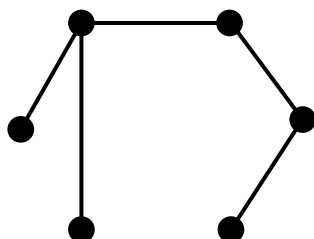
6.3 (8)



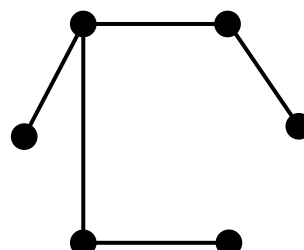
Nodusse 1, 1, 1, 1, 3, 3



Nodusse 1, 1, 1, 1, 2, 4



Nodusse 1, 1, 1, 2, 2, 3



Nodusse 1, 1, 1, 2, 2, 3

[12]

**Totaal vir Module 4: 100 punte**

**Totaal: 100 punte**